# Domain-wall brane model building

## From chirality to cosmology

Damien George

Theoretical Particle Physics group
School of Physics
University of Melbourne
Australia

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THE UNIVERSITY OF MELBOURNE

# Acknowledgements


- Supervisor: Ray Volkas.
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# PhD summary

Focus of PhD: theory of domain-wall brane models, with applications to the standard model of particle physics and cosmology.

## Papers published:

- G. Dando, A. Davidson, D.P. George, R.R. Volkas and K.C. Wali
  *The clash of symmetries in a Randall-Sundrum-like spacetime*
  Phys.Rev. D 72 045016 (2005), arXiv:hep-ph/0507097
- D.P. George and R.R. Volkas
  *Stability of domain walls coupled to Abelian gauge fields*
  Phys.Rev. D 72 105011 (2005), arXiv:hep-ph/0508206
- E. Di Napoli, D. George, M. Hertzberg, F. Metzler and E. Siegel
  *Dark Matter In Minimal Trinification*
  Proceedings of the LXXXVI Les Houches Summer School, pp 517–524 (2006), arXiv:hep-ph/0611012
- D.P. George and R.R. Volkas
  *Kink modes and effective four dimensional fermion and Higgs brane models*
  Phys. Rev. D 75 105007 (2007), arXiv:hep-ph/0612270
- R. Davies and D.P. George
  *Fermions, scalars and Randall-Sundrum gravity on domain-wall branes*
  Phys. Rev. D 76 104010 (2007), arXiv:0705.1391
- R. Davies, D.P. George and R.R. Volkas
  *The standard model on a domain-wall brane?*
  Phys. Rev. D 77 124038 (2008), arXiv:0705.1584
- A. Davidson, D.P. George, A. Kobakhidze, R.R. Volkas and K.C. Wali
  *$SU(5)$ grand unification on a domain-wall brane from an $E_6$-invariant action*
  Phys. Rev. D 77 085031 (2008), arXiv:0710.3432
- D.P. George, M. Trodden, R.R. Volkas
  *Domain-wall brane cosmology*
  In preparation

# Talk outline

Context: physics beyond the standard model – extra dimensions.

We will cover:

- Randall-Sundrum 2 model with delta-function brane.
- Domain walls (kinks/solitons).
- Trapping scalar and fermion fields to a domain wall.
- Using Dvali-Shifman mechanism to trap gauge fields.
- $SU(5)$ grand unified domain-wall model.
- Extending $SU(5)$ to $E_6$.
- Cosmology – the expansion of a brane universe.

# Introduction

We are inspired by the Randall-Sundrum warped metric solution.

RS1 is a compact extra dimension: provides a solution to the hierarchy problem – lots of work on this model. Branes are string theory like objects. Warped throats, inflation, dark matter, ...

(Randall & Sundrum, PRL83, 3370 (1999))

RS2 is an infinite extra dimension: solves the trapping of low-energy gravity. Not as much interest because it doesn't solve any major problems, just introduces another dimension.

(Randall & Sundrum, PRL83, 4690 (1999))

We will pursue RS2 because it seems a natural extension of 3+1 space.

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- take the standard model and general relativity
- add an *infinite* extra space dimension
- recover the standard model and general relativity at low energies

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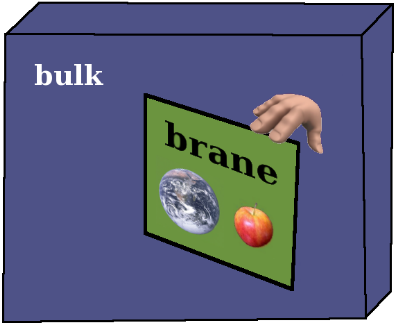


$$\mathcal{S} = \int d^4x \int dy \sqrt{|g|} \left[ -M^3 R + \delta(y) \mathcal{L}_{\mathsf{SM}} \right]$$

# RS2 model

Need brane and bulk sources:

$$\mathcal{S} = \int d^4x \int dy \left[ \sqrt{|g|}(-M^3 R - \Lambda_{\mathsf{bulk}}) + \sqrt{|g^{(4)}|}\delta(y)(\mathcal{L}_{\mathsf{SM}} - \Lambda_{\mathsf{brane}}) \right] \quad (1)$$

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Solve the theory:

- Randall-Sundrum metric ansatz: $ds^2 = e^{-2k|y|} g^{(4)}_{\mu\nu} dx^\mu dx^\nu - dy^2$
- Solve Einstein's equations ($\mathcal{L}_{\text{SM}} = 0$ and $R^{(4)} = 0$):

$$\Lambda_{\text{bulk}} = -12k^2 M^3 \qquad \Lambda_{\text{brane}} = 12kM^3$$

- Write $R$ in terms of $R^{(4)}$: $R = e^{2k|y|} R^{(4)} - 16k\delta(y) + 20k^2$

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Substitute into (1) and integrate over $y$:

$$\mathcal{S} = \int d^4x \sqrt{|g^{(4)}|} \left[ -\frac{M^3}{k} R^{(4)} + \mathcal{L}_{\mathsf{SM}} \right]$$

A dimensionally reduced theory.

# Newton's law

Just need to check Newton's law. Linear tensor fluctuations are:

$$g_{\mu\nu} = e^{-2k|y|}\eta_{\mu\nu} + \sum_n h^{(4)}_{n\ \mu\nu}(x^\mu)\psi_n(y)$$

The zero mode $h^{(4)}_{0\ \mu\nu}$ dominates the Kaluza-Klein tower.

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Newton's law is modified to:

$$V(r) = -G_N \frac{m_1 m_2}{r}\left(1 + \frac{\epsilon^2}{r^2}\right) \qquad \text{(where } \epsilon = 1/k\text{)}$$

Current experimental bounds are very weak:

$$\epsilon < 12\mu\text{m} \quad \Longrightarrow \quad k > 16 \times 10^{-3}\text{eV}$$

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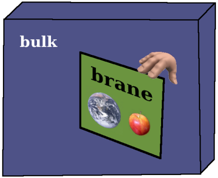
- We have what we wanted: a 5D theory that at low energies looks like our 4D universe.
- But almost no new phenomenology.
- Next step: brane forms naturally.



# Thick and smooth RS2

We want to remove the $\delta(y)$ part of the action:

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Everything from now on is one way of doing that.

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Geometry, hence gravity, is 5D. So why not try to make all fields 5D?

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- Then we show how to trap scalars, fermions and gauge fields to the brane.
- Finally we present a 4+1-d $SU(5)$ based extension to the standard model.

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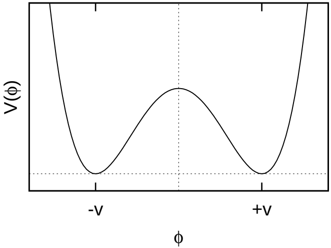
Turn off warped gravity for now – just think about trapping 5D fields.

# A domain wall as a brane




A *domain-wall* is a (thin) region separating two vacua.

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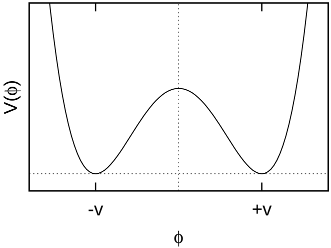


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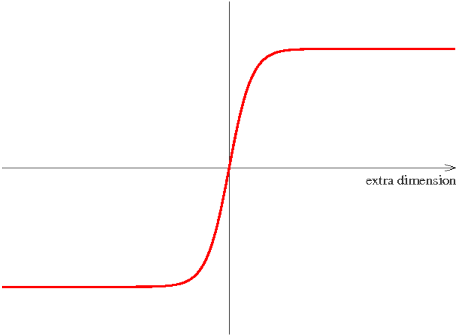
Lagrangian for $\phi(x^\mu, y)$:

$$\mathcal{L} = \frac{1}{2}\partial_M\phi\,\partial^M\phi - V(\phi)$$

A solution is the *kink*:

$$\phi(y) = v\tanh(\sqrt{2\lambda}vy)$$

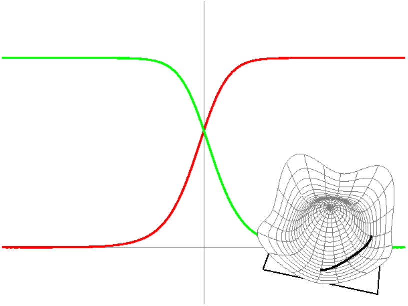
It is stable!



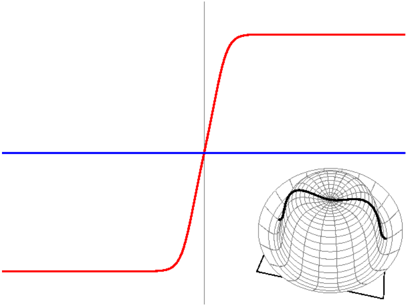
# More complicated domain walls

(DPG & Volkas, PRD72, 105011 (2005))

These examples use two scalar fields to form the wall.



$$V = \lambda_1(\phi_1^2 + \phi_2^2 - v^2)^2 + \lambda_2\phi_1^2\phi_2^2$$



$$V = \lambda(\phi_1^2+\phi_2^2-v_1^2)^2(\phi_1^2+\phi_2^2+v_2^2)$$

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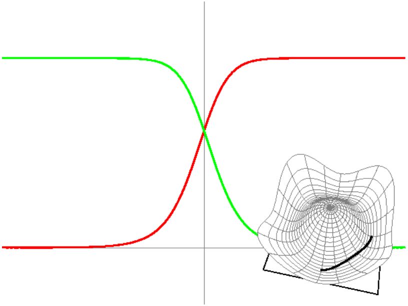

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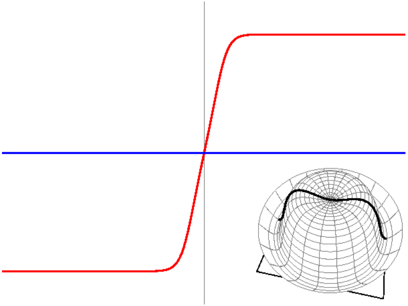

$V = \lambda(\phi_1^2+\phi_2^2-v_1^2)^2(\phi_1^2+\phi_2^2+v_2^2)$

In a realistic model, symmetries dictate $V$.

To determine stability, expand $\phi$ in normal modes about the background: $\phi(x,t) = \phi_{\mathsf{bg}}(x) + \sum_n \xi_n(x)e^{i\omega_n t}$. Make sure $\omega_n^2 \geq 0$

# Trapping matter fields

# Trapping scalar fields

(DPG & Volkas, PRD75, 105007 (2007))

Aim: to trap a 5D scalar field $\Xi(x^\mu, y)$ to the brane.

A simple quartic coupling works:

$$\mathcal{S} = \int d^4x \int dy \left[ \frac{1}{2}\partial^M\phi \, \partial_M\phi - V(\phi) + \frac{1}{2}\partial^M\Xi \, \partial_M\Xi - W(\Xi) - g\phi^2\Xi^2 \right]$$

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Expand $\Xi$ in extra dimensional (Kaluza-Klein) modes:

$$\Xi(x^\mu, y) = \sum_n \xi_n(x^\mu) k_n(y)$$

$\xi_n$ are the 4D fields, $k_n$ their extra-dimensional profile. The profiles satisfy a Schrödinger equation:

$$\left( -\frac{d^2}{dy^2} + 2g\phi_{\mathsf{bg}}^2 \right) k_n(y) = E_n^2 k_n(y)$$

The energy eigenvalues $E_n$ are related to the mass of the 4D field $\xi_n$.

# Trapping via a potential well

(DPG & Volkas, PRD75, 105007 (2007))

The effective potential acts like a well.

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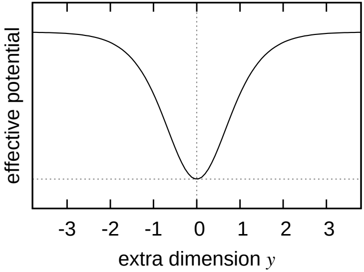


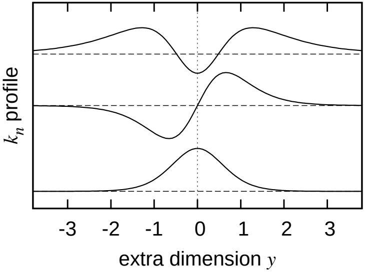

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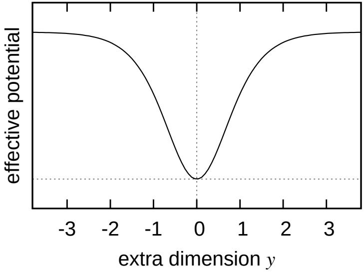


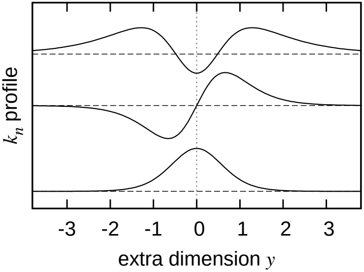


To get 4D theory, substitute mode expansion into action and integrate $y$:

$$\mathcal{S} = \int d^4x \left[ \sum_n \left( \frac{1}{2} \partial^\mu \xi_n \partial_\mu \xi_n - m_n^2 \xi_n^2 \right) + (\text{higher order terms}) \right]$$

- Orthonormal basis $k_n \implies$ diagonal kinetic and mass terms.
- $m_n$ can be tuned.

# Trapping fermions



We can trap a fermion $\Psi(x^\mu, y)$ to the brane with a Yukawa coupling:

$$\mathcal{S} = \int d^4x \int dy \left[ \frac{1}{2}\partial^M\phi \, \partial_M\phi - V(\phi) + \overline{\Psi} i \Gamma^M \partial_M \Psi - h\phi\overline{\Psi}\Psi \right]$$

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Decompose into left- and right-chiral fields and Kaluza-Klein modes:

$$\Psi(x^\mu, y) = \sum_n \left[ \psi_{Ln}(x^\mu) f_{Ln}(y) + \psi_{Rn}(x^\mu) f_{Rn}(y) \right]$$

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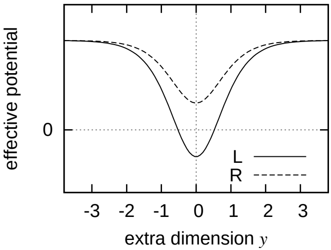
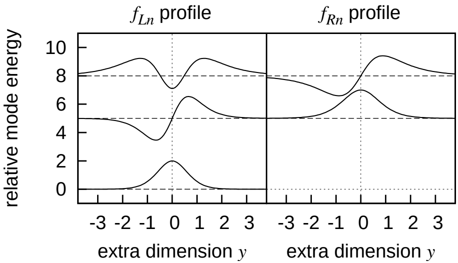
Schrödinger equation (mode index $n$ suppressed):

$$\left( -\frac{d^2}{dy^2} + (h^2\phi_{\text{bg}}^2 \mp h\phi'_{\text{bg}}) \right) f_{L,R}(y) = m^2 f_{L,R}(y)$$



# Gravity and matter fields

Brane (domain wall/kink), trapped scalar and fermion. Plus gravity:

$$\mathcal{S} = \int d^4x \int dy \sqrt{|g|} \Big[ - M^3 R - \Lambda_{\text{bulk}} + \frac{1}{2}\partial^M \phi \, \partial_M \phi - V(\phi)$$
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Dimensionally reduce by integrating over $y$:

$$\mathcal{S} = \int d^4x \sqrt{|g^{(4)}|} \Big[ - M_{\text{4D}}^2 R^{(4)} + (\text{brane dynamics})$$
$$+ \frac{1}{2}\partial^\mu \xi_n \partial_\mu \xi_n - m_n^2 \xi_n^2 - \tau_{mnop} \xi_m \xi_n \xi_o \xi_p - (\text{brane interactions})$$
$$+ \overline{\psi}_{L0} i \gamma^\mu \partial_\mu \psi_{L0} + \overline{\psi}_n (i\gamma^\mu \partial_\mu - \mu_n) \psi_n - (\text{brane interactions}) \Big]$$

4D parameters ($M_{\text{4D}}$, $m_n$, $\tau_{mnop}$, $\mu_n$, brane dynamics) determined by eigenvalue spectra and overlap integrals.

# Warped matter

(Davies & DPG, PRD76, 104010 (2007))

Warped metric $ds^2 = e^{-2\sigma(y)}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2$ modifies profile equation:

$$\left(-\frac{d^2}{dy^2} + 5\sigma'\frac{d}{dy} + 2\sigma'' - 6\sigma'^2 + U(y)\right) f_{Ln}(y) = m_n^2 e^{2\sigma} f_{Ln}(y)$$

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Conformal coordinates $ds^2 = e^{-2\sigma(y(z))}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2)$.
Rescale $f_{Ln}(y) = e^{2\sigma}\tilde{f}_{Ln}(z)$:

$$\left(-\frac{d^2}{dz^2} + e^{-2\sigma(y(z))}U(y(z))\right)\tilde{f}_{Ln}(z) = m_n^2 \tilde{f}_{Ln}(z)$$

Matter trapping potentials are warped down.

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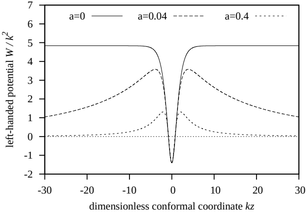
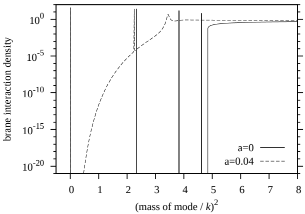
Matter trapping potentials are warped down.

- Finite bound state lifetimes.
- Resonances.
- Tiny probability of interaction with continuum.




($a \sim 1/M^3 \sim$ 5D Newton's constant)

# Trapping gauge fields

# Confining gauge fields

Need to trap gauge fields or e.g. Coulomb potential would be $V_{\text{Coulomb}} \sim 1/r^2$.

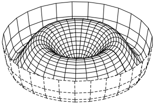
Not as simple as a Kaluza-Klein mode expansion:

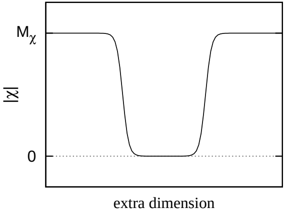
- Photon and gluons *must* remain massless.
- Need to preserve gauge universality at 3+1-d level.

We use the Dvali-Shifman mechanism, Dvali & Shifman, PLB396, 64 (1997).

The following discussion is based on Dvali, Nielsen & Tetradis, PRD77, 085005 (2008).

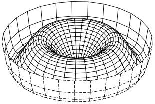
$U(1)$ gauge theory (think 5d photon).
Charged Higgs $\chi$ (gives mass to photon).

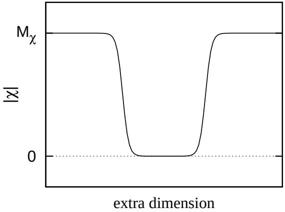


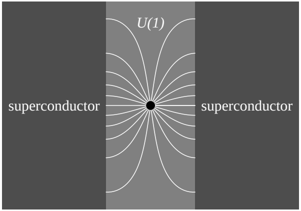


# Abelian Higgs model

$U(1)$ gauge theory (think 5d photon).
Charged Higgs $\chi$ (gives mass to photon).






In the bulk:

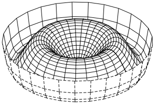
- $U(1)$ is broken, massive photon $\sim M_\chi$.
- Higgs vacuum is a superconductor.
- Electric charges are screened.

On the brane:

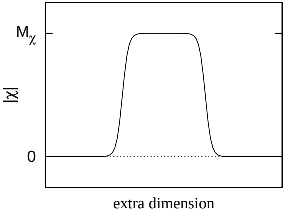
- $U(1)$ is restored, massless photon.
- Electric field ends on Higgs vacuum.

Charge screening leaks onto the brane!

# Using a dual superconductor



$SU(2)$ gauge theory (3 “gluons”).
Adjoint Higgs $\chi^a$ $(a = 1, 2, 3)$ (gives mass).



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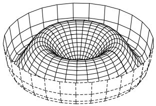

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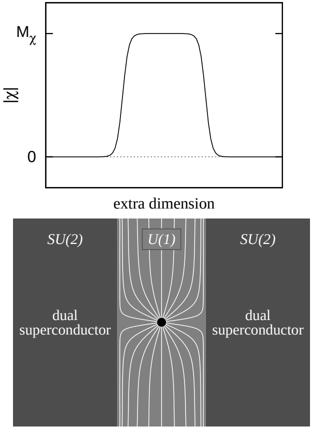



In the bulk:

- $SU(2)$ is restored, in confining regime.
- Large mass gap $\sim M_\chi$ to colourless state.
- QCD-like vacuum is dual superconductor.

On the brane:

- $SU(2)$ broken to $U(1)$, massless photon.
- Electric field repelled from dual superconductor.

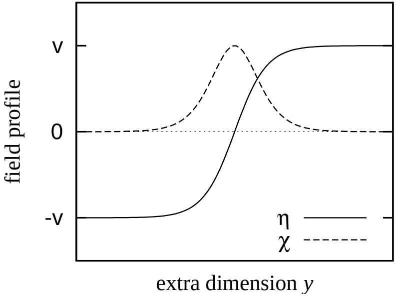
For distances much larger than brane width, electric potential $\sim 1/r$.

# Dvali-Shifman model

Stabilise the domain wall with an extra uncharged scalar field $\eta$:

$$\mathcal{S} = \int d^4x \int dy \Big[ \frac{-1}{4g^2} G^{aMN} G^a_{MN} + \frac{1}{2}\partial^M \eta \partial_M \eta + \frac{1}{2}(D^M \chi^a)^\dagger D_M \chi^a - \lambda(\eta^2 - v^2)^2 - \frac{\lambda'}{2}(\chi^a\chi^a + \kappa^2 - v^2 + \eta^2)^2 \Big]$$

- $\eta$ has a kink profile.
- If $\kappa^2 - v^2 < 0$, $\chi$ becomes tachyonic near domain wall (where $\eta \sim 0$).
- True vacuum has $\chi \neq 0$ near domain wall.
- $\chi$ breaks symmetry near wall and confines gauge fields.



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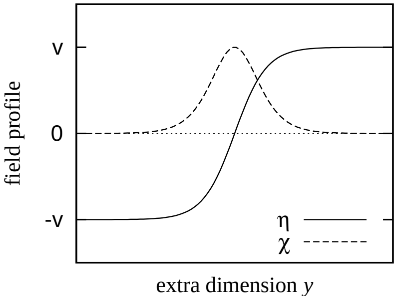


Can add gravity: self consistently solve $\sigma$ (warped metric profile), $\eta$, $\chi$.

The Dvali-Shifman mechanism:

- Works with any non-Abelian $SU(N)$ theory.
- Assumes the $SU(N)$ theory is confining (not proven for 5D).
- Has gauge universality:
    - Charges in the bulk are connected to the brane by a flux tube.
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Obvious choice for $SU(N)$ group is $SU(5)$.

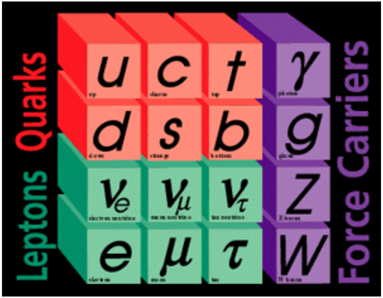
# $SU(5)$ basics

# Quantum numbers of the standard model

Representations under $SU(3)\times SU(2)_L\times U(1)_Y$:

$$q_L \sim (\mathbf{3},\mathbf{2})_{1/3} \quad u_R \sim (\mathbf{3},\mathbf{1})_{4/3} \quad d_R \sim (\mathbf{3},\mathbf{1})_{-2/3}$$

$$l_L \sim (\mathbf{1},\mathbf{2})_{-1} \quad \nu_R \sim (\mathbf{1},\mathbf{1})_{0} \quad e_R \sim (\mathbf{1},\mathbf{1})_{-2}$$



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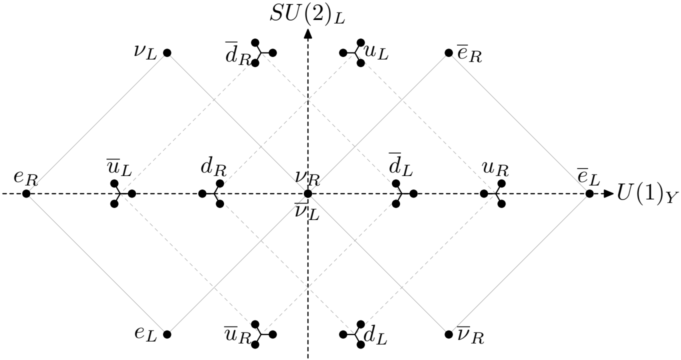

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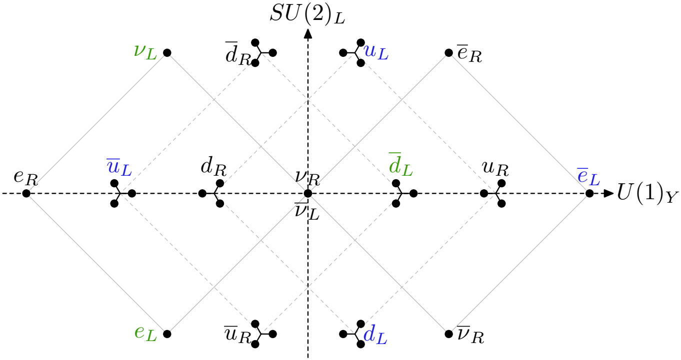


$$\mathbf{5}^* \supset \overline{d}_L^{r,w,b} \quad \nu_L \quad e_L$$
$$(\mathbf{5}\times\mathbf{5})_A = \mathbf{10} \supset \overline{u}_L^{r,w,b} \quad u_L^{r,w,b} \quad d_L^{r,w,b} \quad \overline{e}_L$$

# Putting it all together

# The $SU(5)$ model

(Davies, DPG & Volkas, PRD77, 124038 (2008))

Want the standard model on the brane: $SU(3) \times SU(2)_L \times U(1)_Y$.

Dvali-Shifman needs a *larger* gauge group in the bulk:

$SU(5)$ is a perfect fit!

Unify the fermions as usual: $\mathbf{5}^*$, $\mathbf{10}$.
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Summary:

- $4+1$-dimensional theory – all spatial dimensions the same.
- $SU(5)$ local gauge symmetry, $\mathbb{Z}_2$ discrete symmetry.
- Field content:
  - gauge fields: $G_{MN} \sim \mathbf{24}$.
  - scalars: $\eta \sim \mathbf{1}$, $\chi \sim \mathbf{24}$, $\Phi \sim \mathbf{5}^*$.
  - fermions: $\Psi_5 \sim \mathbf{5}^*$, $\Psi_{10} \sim \mathbf{10}$.
- The standard model emerges as a low energy approximation.

Ignore gravity for now.

# The action (without gravity)

(Davies, DPG & Volkas, PRD77, 124038 (2008))

The theory is described by:

$$\mathcal{S} = \int d^4x \int dy \Big[ \frac{-1}{4g^2} G^{aMN} G^a_{MN} + \frac{1}{2}\partial^M \eta \partial_M \eta + \mathrm{Tr}\left( (D^M\chi)^\dagger (D_M \chi) \right)$$
$$+ (D^M\Phi)^\dagger (D_M\Phi) + \overline{\Psi}_5 i\Gamma^M D_M \Psi_5 + \overline{\Psi}_{10} i\Gamma^M D_M \Psi_{10}$$
$$- h_{5\eta}\overline{\Psi}_5\Psi_5\eta - h_{5\chi}\overline{\Psi}_5\chi^T\Psi_5$$
$$- h_{10\eta}\,\mathrm{Tr}(\overline{\Psi}_{10}\Psi_{10})\eta + 2h_{10\chi}\,\mathrm{Tr}(\overline{\Psi}_{10}\chi\Psi_{10})$$
$$- h_-\overline{(\Psi_5)^c}\Psi_{10}\Phi - h_+(\epsilon\overline{(\Psi_{10})^c}\Psi_{10}\Phi^*) + \text{h.c.}$$
$$- (c\eta^2 - \mu_\chi^2)\,\mathrm{Tr}(\chi^2) - d\eta\,\mathrm{Tr}(\chi^3)$$
$$- \lambda_1 \left[\mathrm{Tr}(\chi^2)\right]^2 - \lambda_2\,\mathrm{Tr}(\chi^4) - l(\eta^2 - v^2)^2$$
$$- \mu_\Phi^2\Phi^\dagger\Phi - \lambda_3(\Phi^\dagger\Phi)^2 - \lambda_4\Phi^\dagger\Phi\eta^2$$
$$- 2\lambda_5\Phi^\dagger\Phi\,\mathrm{Tr}(\chi^2) - \lambda_6\Phi^\dagger(\chi^T)^2\Phi - \lambda_7\Phi^\dagger\chi^T\Phi\eta \Big]$$

with kinetic, brane trapping, mass and Dvali-Shifman terms.

# Split fermions

(Davies, DPG & Volkas, PRD77, 124038 (2008))

Let $\Psi_{nY}$ be the components of $\Psi_5$ and $\Psi_{10}$ ($n = 5, 10$, $Y$ = hypercharge of component), e.g. $\Psi_5 \supset \Psi_{5,-1} = l_L$. Dirac equation:

$$\left[ i\Gamma^M \partial_M - h_{n\eta}\eta(y) - \sqrt{\frac{3}{5}}\frac{Y}{2} h_{n\chi}\chi_1(y) \right] \Psi_{nY}(x^\mu, y) = 0$$

Each $\Psi_{nY}$ is a non-chiral 5D field: need to extract the confined left-chiral zero-mode (recall the mode expansion and Schrödinger equation approach):

$$\Psi_{nY}(x^\mu, y) = \psi_{nY,L}(x^\mu) f_{nY}(y) + \text{massive modes}$$

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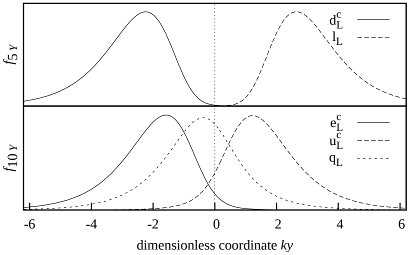
The effective Schrödinger potential depends on $Y$.

Thus each component $\psi_{nY,L}$ has a different profile $f_{nY}$.



# Split Higgs

(Davies, DPG & Volkas, PRD77, 124038 (2008))

$\Phi$ contains the Higgs doublet $\Phi_w$ and a coloured triplet $\Phi_c$. Mode expand $\Phi_{w,c}(x^\mu, y) = \phi_{w,c}(x^\mu) p_{w,c}(y)$. Schrödinger equation for $p_{w,c}$ is:

$$\left( -\frac{d^2}{dy^2} + \frac{3Y^2}{20}\lambda_6 \chi_1^2 + \sqrt{\frac{3}{5}}\frac{Y}{2}\lambda_7 \eta \chi_1 + \ldots \right) p_{w,c}(y) = m_{w,c}^2 p_{w,c}(y)$$

*Critical* that ground states have:

- $m_w^2 < 0$ to break electroweak symmetry.
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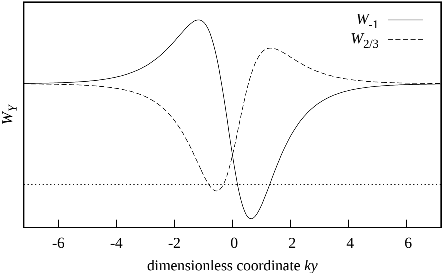
Large enough parameter space to allow this.



# Features

(Davies, DPG & Volkas, PRD77, 124038 (2008))

Standard model parameters are computed from overlap integrals.

With one generation of fermions, parameters are easy to fit.

The model overcomes the major $SU(5)$ obstacles:

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Adding gravity:

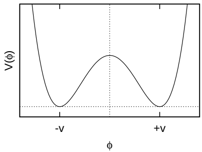
- Solve for warped metric, kink and Dvali-Shifman background.
- Continuum fermion and scalar modes are highly suppressed on the brane.
- Main features remain.

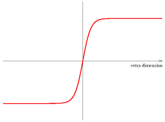
# Going beyond $SU(5)$

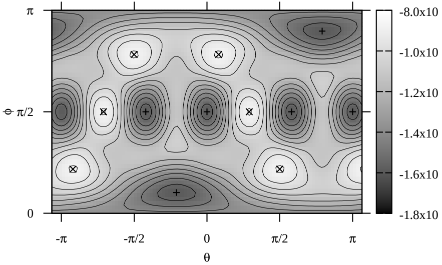
(Davidson, DPG, Kobakhidze, Volkas & Wali, PRD77, 085031 (2008))

One promising extension is to the $E_6$ group:

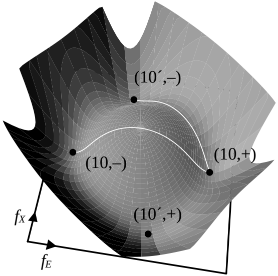
- $E_6 \to SO(10)$ in the bulk.
- $SO(10) \to SU(5)$ on the brane due to clash-of-symmetries (CoS) and Dvali-Shifman.
- Can eliminate kink scalar field $\eta$.
- Can unify $\Psi_5$ and $\Psi_{10}$.
- Large reduction of free parameters.




(+'s are $SO(10)$'s)




(top domain-wall has CoS)

# Domain-wall cosmology

# The scale factor on a brane

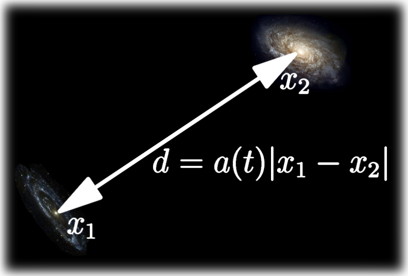
Most important cosmological fact: expanding universe.

FRW: $ds^2 = -dt^2 + a^2(t)\, d\mathbf{x} \cdot d\mathbf{x}$

Energy density $\rho$ dictates expansion:

$$H = \frac{\dot{a}}{a} \qquad\qquad H^2 = \frac{8\pi G}{3}\rho$$

(Hubble parameter) (spatially flat universe)



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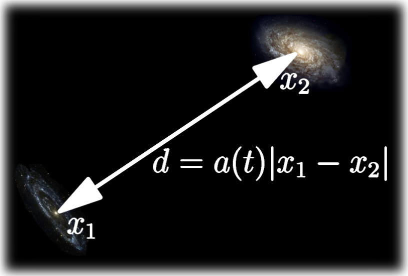



FRW for a brane: $ds^2 = -n^2(t,y)dt^2 + a^2(t,y)\, d\mathbf{x} \cdot d\mathbf{x} + dy^2$

$$H^2_{\text{brane}} = \frac{8\pi G}{3}\left(1 + \frac{\rho}{2\sigma}\right)\rho$$

$\sigma$ is the energy density (tension) of the brane; $\sigma \gg (1MeV)^4$ from BBN.

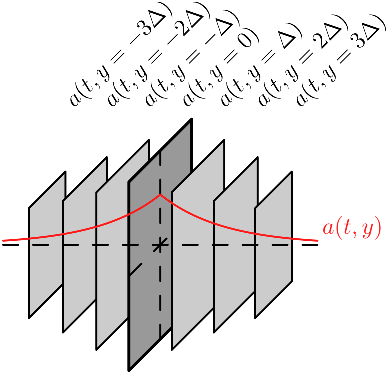
(Binétruy, Deffayet & Langlois, Nucl. Phys. B565, 269 (2000))

# Species-dependent scale factor

(DPG, Trodden & Volkas, in prep.)

$$ds^2 = -n^2(t,y)dt^2 + a^2(t,y)d\mathbf{x} \cdot d\mathbf{x} + dy^2$$

Each slice at constant $y$ has a different scale factor.



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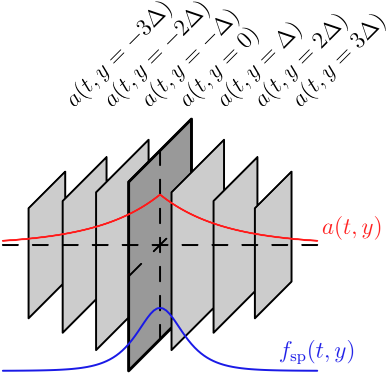


For thick branes, different species (electron, quark, KK mode) experience different expansion rates.

$$a_{\text{sp}}(t) = a_0(t)\left(1 + \frac{\dot{\rho}}{2\sigma H_0} I_{\text{sp}}(t)\right)$$

$$I_{\text{sp}}(t) = \frac{\int f_{\text{sp}}^2(t,y)\, dy}{\int f_{\text{sp}}^2(t,y)\, e^{-\sigma|y|/6M_5^3} dy} - 1$$

Different localisation profile $f_{\text{sp}}$ for different energies! No chiral fermions!

# Summary and outlook

Our universe may be a brane residing in a higher dimensional bulk.

We have investigated the possibility that the brane is in fact a domain-wall.

Main results:

- Scalar field: forms stable domain wall.
- RS2 warped metric: traps gravity.
- Dvali-Shifman mechanism: traps gauge fields.
- $SU(5)$ domain-wall model: overcomes major $SU(5)$ obstacles.
- $E_6$ extension: unify fields and reduce parameters.
- Cosmology: species-dependent expansion rate.

Outlook: extra-dimensions a real possibility!
Will be tested by LHC (e.g. KK modes) and cosmological observations.