Model building using Lie-point symmetries

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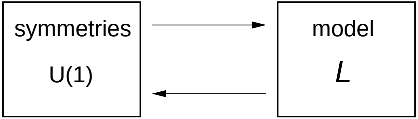
University of Zurich / ETH — 11$^{\text{th}}$ October 2011

(arXiv:1105.4604)

Overview



Systematically find all symmetries of a model,

$\rightarrow$ even if symmetry is spontaneously broken,

$\rightarrow$ also derive parameter relationships that give enhanced symmetries.

Overview:

- The Lie point symmetry method.
- Examples: 2 scalars, N scalars.
- Action versus equations of motion.
- Spontaneous symmetry breaking.
- Gauge symmetries.
- Automation.
- The standard model.

The Lie point symmetry method

The Lie point symmetry method consists of finding the determining equations, whose solutions describe infinitesimal symmetries, and then solving these equations.

"Point": transformations depend only on coords and fields, not on derivatives of fields.

1. Derive the determining equations of the system.
   Coordinates x^μ and fields ϕ_i;
   infinitesimal variations $\delta x^\mu = \eta^\mu(x,\phi)$ and $\delta\phi_i = \chi_i(x,\phi)$.
2. Solve the determining equations, or at least reduce to a standard form. Solution set can branch depending on parameter values (e.g. $m = 0$, $m_1 = m_2$).
3. Compute the rank (number of generators) of the symmetry set(s).
   $R = (N_{\text{const}}, N_{f_1}(\lambda), N_{f_2}(\lambda_1,\lambda_2),\ldots)$.
4. (Optional) Compute the action of the symmetries.

The method

The LPS method is general and powerful:

- an exhaustive search of continuous symmetries;
- yields all interesting relationships between parameters;
- finding the rank is guaranteed to terminate in finite time, determined by the number of coordinates and number of fields;
- applicable to any set of differential equations (coordinates=independent-variables, fields=dependent-variables).

Two ways to derive the determining equations.

Variation of the action

Infinitesimal Lie point symmetries:

$$x^\mu \to x^\mu + \eta^\mu(x,\phi) \qquad \phi_i \to \phi_i + \chi_i(x,\phi)$$

$S \to S + \delta S$ should be unchanged.

Solve for the fields $\to$ Euler-Lagrange equations: $\frac{\partial \mathcal{L}}{\partial \phi_i} - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_i)} \right) = 0.$

Form a divergence $\to$ Noether's theorem: $\partial_\mu \left[\mathcal{L}\eta^\mu + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_i)} (\chi_i - \eta^\nu \partial_\nu \phi_i) \right] = 0.$

Solve for the infinitesimals $\to$ master determining equation:

$$\boxed{\mathcal{L}\frac{\mathrm{d}\eta^\mu}{\mathrm{d}x^\mu} + \frac{\partial \mathcal{L}}{\partial x^\mu}\eta^\mu + \frac{\partial \mathcal{L}}{\partial \phi_i}\chi_i + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_i)} \left(\frac{\mathrm{d}\chi_i}{\mathrm{d}x^\mu} - \frac{\partial \phi_i}{\partial x^\nu}\frac{\mathrm{d}\eta^\nu}{\mathrm{d}x^\mu} \right) = 0}$$

Total derivative: $\frac{\mathrm{d}}{\mathrm{d}x^\mu} \equiv \frac{\partial}{\partial x^\mu} + \frac{\partial \phi_i}{\partial x^\mu}\frac{\partial}{\partial \phi_i}.$

Example: two scalars

Only field symmetries, $\phi_i \to \phi_i + \chi_i(\phi_i)$.
Master determining equation:

$$\frac{\partial \mathcal{L}}{\partial \phi_i}\chi_i + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_i)}\frac{\partial \phi_j}{\partial x^\mu}\frac{\partial \chi_i}{\partial \phi_j} = 0\,.$$

Apply to Lagrangian

$$\mathcal{L} = \frac{1}{2}\partial^\mu\phi_1\partial_\mu\phi_1 + \frac{1}{2}\partial^\mu\phi_2\partial_\mu\phi_2 - \frac{1}{2}m_1^2\phi_1^2 - \frac{1}{2}m_2^2\phi_2^2\,.$$

Determining equation is

$$\begin{aligned}- m_1^2\phi_1\chi_1 - m_2^2\phi_2\chi_2 + \partial^\mu\phi_1\partial_\mu\phi_1\frac{\partial\chi_1}{\partial\phi_1} \\ + \partial^\mu\phi_1\partial_\mu\phi_2\frac{\partial\chi_1}{\partial\phi_2} + \partial^\mu\phi_2\partial_\mu\phi_1\frac{\partial\chi_2}{\partial\phi_1} + \partial^\mu\phi_2\partial_\mu\phi_2\frac{\partial\chi_2}{\partial\phi_2} = 0\,.\end{aligned}$$

Equate independent terms to zero:

$$-m_1^2\phi_1\chi_1 - m_2^2\phi_2\chi_2 = 0\,,\quad \frac{\partial\chi_1}{\partial\phi_1} = 0\,,\quad \frac{\partial\chi_1}{\partial\phi_2} + \frac{\partial\chi_2}{\partial\phi_1} = 0\,,\quad \frac{\partial\chi_2}{\partial\phi_2} = 0\,.$$

Example: two scalars

Determining equations:

$$-m_1^2\phi_1\chi_1 - m_2^2\phi_2\chi_2 = 0\,, \quad \frac{\partial\chi_1}{\partial\phi_1} = 0\,, \quad \frac{\partial\chi_1}{\partial\phi_2} + \frac{\partial\chi_2}{\partial\phi_1} = 0\,, \quad \frac{\partial\chi_2}{\partial\phi_2} = 0\,.$$

General solution to last three equations:

$$\chi_1(\phi_2) = \alpha_1 + \beta\phi_2\,, \qquad \chi_2(\phi_1) = \alpha_2 - \beta\phi_1\,.$$

Symmetries:

- α_1: shift of ϕ_1.
- α_2: shift of ϕ_2.
- β: rotation between ϕ_1 and ϕ_2.

Final determining equation is

$$\alpha_1 m_1^2\phi_1 + \alpha_2 m_2^2\phi_2 + \beta(m_1^2 - m_2^2)\phi_1\phi_2 = 0\,.$$

$\rightarrow$ *the model parameters dictate the symmetries.*

Example: two scalars

Recall the Lagrangian

$$\mathcal{L} = \frac{1}{2}\partial^\mu \phi_1 \partial_\mu \phi_1 + \frac{1}{2}\partial^\mu \phi_2 \partial_\mu \phi_2 - \frac{1}{2} m_1^2 \phi_1^2 - \frac{1}{2} m_2^2 \phi_2^2 \,.$$

General solution for symmetries

$$\begin{aligned} \phi_1 \to \bar{\phi}_1 \quad \text{with} \quad \bar{\phi}_1' = \chi_1 = \alpha_1 + \beta \bar{\phi}_2 \\ \phi_2 \to \bar{\phi}_2 \quad \text{with} \quad \bar{\phi}_2' = \chi_2 = \alpha_2 - \beta \bar{\phi}_1 \end{aligned}$$

Final algebraic determining equation

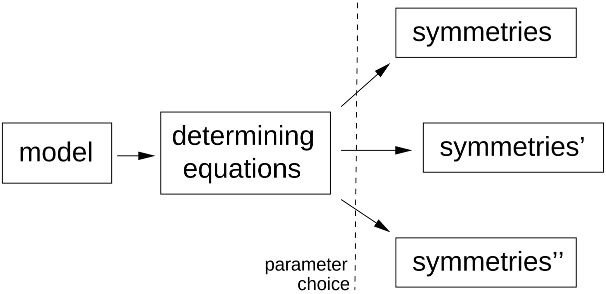
$$\alpha_1 m_1^2 \phi_1 + \alpha_2 m_2^2 \phi_2 + \beta (m_1^2 - m_2^2) \phi_1 \phi_2 = 0 \,.$$

- $m_1 = 0$ allows $\alpha_1 \neq 0$, symmetry $\bar{\phi}_1 = \phi_1 + \alpha_1 \epsilon$.
- $m_2 = 0$ allows $\alpha_2 \neq 0$, symmetry $\bar{\phi}_2 = \phi_2 + \alpha_2 \epsilon$.
- $m_1^2 = m_2^2$ allows $\beta \neq 0$. The symmetry is

$$\begin{pmatrix} \bar{\phi}_1 \\ \bar{\phi}_2 \end{pmatrix} = \begin{pmatrix} \cos \beta\epsilon & \sin \beta\epsilon \\ -\sin \beta\epsilon & \cos \beta\epsilon \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \,.$$

Solving the determining equations

We have seen how to find and solve the determining equations, and find parameter relationships.



N interacting scalar fields

Symmetries dictated by structure of interactions between fields.

General Lagrangian for N spin-0 fields

$$\mathcal{L} = \tfrac{1}{2}\partial^\mu \phi_i \partial_\mu \phi_i - V(\phi)\,.$$

Determining equations

$$V\partial_\mu \eta^\mu + \frac{\partial V}{\partial \phi_i}\chi_i = 0\,,$$

$$\partial^\mu \chi_i - V\frac{\partial \eta^\mu}{\partial \phi_i} = 0 \qquad \forall \mu\ \forall i\,, \qquad (\chi = \chi(\phi))$$

$$\partial^\mu \eta^\nu + \partial^\nu \eta^\mu = 0 \qquad \forall \mu\ \forall \nu,\ \mu \neq \nu\,, \qquad \text{(Poincaré)}$$

$$\frac{\partial \chi_i}{\partial \phi_j} + \frac{\partial \chi_j}{\partial \phi_i} = 0 \qquad \forall i\ \forall j,\ i \neq j\,, \qquad \text{(shift, rot.)}$$

$$\frac{1}{2}\partial_\sigma \eta^\sigma - \partial_{\bar{\mu}}\eta^{\bar{\mu}} + \frac{\partial \chi_{\bar{i}}}{\partial \phi_{\bar{i}}} = 0 \qquad \forall \bar{\mu}\ \forall \bar{i}\,, \qquad \text{(scaling)}$$

$$\frac{\partial \eta^\mu}{\partial \phi_i} = 0 \qquad \forall \mu\ \forall i\,. \qquad (\eta = \eta(x))$$

N interacting scalar fields

General Lagrangian

$$\mathcal{L} = \tfrac{1}{2}\partial^\mu \phi_i \partial_\mu \phi_i - V(\phi)\,.$$

For $D \neq 2$ the general coordinate symmetries are ($b^{\mu\nu}$ anti-symm)

$$\eta^\mu(x) = a^\mu + b^\mu{}_\nu x^\nu + c\, x^\mu\,.$$

General field symmetries are (β_{ij} anti-symm)

$$\chi_i(\phi) = \alpha_i + \beta_{ij}\phi_j + \frac{2-D}{2} c\, \phi_i\,.$$

Remaining determining equation is

$$DcV + \frac{\partial V}{\partial \phi_i}\left(\alpha_i + \beta_{ij}\phi_j + \frac{2-D}{2} c\, \phi_i\right) = 0\,.$$

Form of V $\leftrightarrow$ allowed symmetries.

Symmetries of one scalar

Specialise to $N = 1$:

$$DcV + \frac{\mathrm{d}V}{\mathrm{d}\phi}\left(\alpha + \frac{2-D}{2}c\,\phi\right) = 0\,.$$

Four distinct cases:

$V = 0$: α and c free. Independent shift and scale symmetries.
Rank associated with field is $R_\chi = (2)$.

$V = \text{const}$: $c = 0$ but α free.
Field rank $R_\chi = (1)$.

$V = \lambda(\phi + v)^{\frac{2D}{D-2}}$: Solve above differential equation.
Given v, relationship between shift and scale symmetry is fixed by $v = 2\alpha/(2-D)c$.
Field rank $R_\chi = (1)$.

V arbitrary: $\alpha = c = 0$. No shift or scale symmetry.
Field rank $R_\chi = (0)$.

Symmetries of two scalars

$$DcV+\frac{\partial V}{\partial \phi_1}\left(\alpha_1+\beta\phi_2+\frac{2-D}{2}c\phi_1\right)+\frac{\partial V}{\partial \phi_2}\left(\alpha_2-\beta\phi_1+\frac{2-D}{2}c\phi_2\right)=0.$$

Go to polar field variables, $\phi_1 = r\cos\theta$, $\phi_2 = r\sin\theta$:

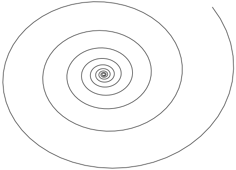
$$\mathcal{L}=\frac{1}{2}\partial^\mu r\partial_\mu r+r^2\frac{1}{2}\partial^\mu\theta\partial_\mu\theta - V(r,\theta)\;.$$

Determining equation is

$$DcV+\frac{\partial V}{\partial r}\left(\alpha_1\cos\theta+\alpha_2\sin\theta+\frac{2-D}{2}cr\right)-\frac{\partial V}{\partial \theta}\left(\alpha_1\frac{\sin\theta}{r}-\alpha_2\frac{\cos\theta}{r}+\beta\right)=0.$$

A solution:

$$V(r,\theta)=\lambda\left(r^k - v\mathrm{e}^{l\theta}\right)^m\;.$$



Parameters related: $mk = 2D/(D-2)$.
Symmetries related: $(2-D)kc = 2l\beta$.
Acts as: $r \to \mathrm{e}^{l\beta/k}r$, $\theta \to \theta+\beta$, $x^\mu \to \mathrm{e}^c x^\mu$.

Equations of motion approach

Distinction between the symmetries of action and symmetries of corresponding equations of motion.

G a symmetry of an action $\implies$ G also a symmetry of the Euler-Lagrange equations. Converse not necessarily true.

Denote the system by $\Delta_j(x^\mu, \phi_i, \partial\phi_i) = 0$.

1. Construct the prolonged symmetry operator $\mathrm{pr}^{(k)}\,\boldsymbol{\alpha}$.

$$\boldsymbol{\alpha} = \eta^\mu \frac{\partial}{\partial x^\mu} + \chi_i \frac{\partial}{\partial \phi_i} \; .$$

   Prolongation extends $\boldsymbol{\alpha}$ to include all possible combinations of derivatives of ϕ, to order k.

2. Apply $\mathrm{pr}^{(k)}\,\boldsymbol{\alpha}$ to the system: $(\mathrm{pr}^{(k)}\,\boldsymbol{\alpha} \cdot \Delta)|_{\Delta=0} = 0$.
3. Equate all independent coefficients to zero $\rightarrow$ determining equations.

Equations of motion approach

For zeroth-order equations:

$$\boldsymbol{\alpha} = \eta^\mu \frac{\partial}{\partial x^\mu} + \chi_i \frac{\partial}{\partial \phi_i} \, .$$

For second-order equations (most relevant for Euler-Lagrange):

$$\mathrm{pr}^{(2)} \, \boldsymbol{\alpha} = \eta^\mu \frac{\partial}{\partial x^\mu} + \chi_i \frac{\partial}{\partial \phi_i} + X_{\mu i} \frac{\partial}{\partial(\partial_\mu \phi_i)} + \sum_{\mu \leq \nu} Y_{\mu\nu i} \frac{\partial}{\partial(\partial_{\mu\nu} \phi_i)} \, ,$$

where

$$X_{\mu i} = D_\mu(\chi_i) - (\partial_\nu \phi_i) D_\mu(\eta^\nu) \, ,$$
$$Y_{\mu\nu i} = D_\mu D_\nu(\chi_i) - (\partial_{\nu\rho}\phi_i) D_\mu(\eta^\rho) - (\partial_{\mu\rho}\phi_i) D_\nu(\eta^\rho) - (\partial_\rho \phi_i) D_\mu D_\nu(\eta^\rho) \, ,$$

and total derivative is

$$D_\mu = \partial_\mu + (\partial_\mu \phi_i) \frac{\partial}{\partial \phi_i} + (\partial_{\mu\nu}\phi_i) \frac{\partial}{\partial(\partial_\nu \phi_i)} + \sum_{\nu \leq \rho} (\partial_{\mu\nu\rho}\phi_i) \frac{\partial}{\partial(\partial_{\nu\rho}\phi_i)} \, .$$

Equations of motion example

System defined by Euler-Lagrange equation $\ddot{\phi} - \phi'' + m^2\phi = 0$.

What are its symmetries?

- $m = 0$ has

$$\eta^t(t,x) = F_+(t+x) + F_-(t-x) \,,$$
$$\eta^x(t,x) = F_+(t+x) - F_-(t-x) + f \,,$$
$$\chi(t,x,\phi) = G_+(t+x) + G_-(t-x) + g\,\phi(t,x) \,.$$

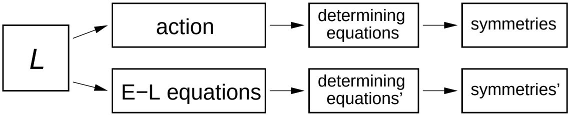
- $m \neq 0$ has

$$\eta^t(x) = a^t + bx \,,$$
$$\eta^x(t) = a^x + bt \,,$$
$$\chi(t,x,\phi) = \int_{-\infty}^{+\infty} dk \left[H_+(k)\, \mathrm{e}^{i(\omega t + kx)} + H_-(k)\, \mathrm{e}^{i(\omega t - kx)} \right] + g\,\phi(t,x) \,,$$

where $\omega = \sqrt{k^2 + m^2}$.

Remarks on the LPS method

The LPS method provides an exhaustive list of symmetries and parameter relationships that yield an enhanced symmetry.



Any spin representation, or even particles that do not respect Lorentz symmetry, can be written in terms of real fields.
Any action can be expanded in terms of its real components.

The LPS method works for all continuous symmetries that depend on the coordinates and fields (but not derivatives of the fields).
Includes local gauge symmetries as well as general relativity.
Supersymmetry: requires introduction of anti-commuting coordinates.

Works for non-linear symmetries and spontaneously broken symmetries.

Non-linear symmetries

Field (no coordinate) symmetries of

$$\mathcal{L} = \phi^m (\partial^\mu \phi \partial_\mu \phi)^n \, .$$

m and $n \neq 0$ are constant exponents.

Action approach, determining equation

$$m\phi^{m-1}\chi + 2n\phi^m \frac{\mathrm{d}\chi}{\mathrm{d}\phi} = 0 \, .$$

Solve for χ:

$$\chi = a\phi^{-m/2n} \qquad a \text{ is integration constant} \, .$$

Non-linear symmetry acts by $\bar{\phi}' = a\bar{\phi}^{-m/2n}$, solution

$$\phi \to (\phi^p + pa\epsilon)^{1/p} \qquad \text{with} \quad p = 1 + m/2n \, .$$

Spontaneously broken symmetries

Spontaneously broken scale symmetry:

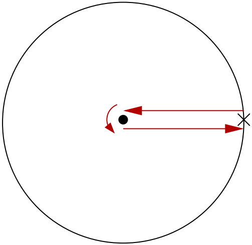
$V = \lambda\phi^4$ has scale symmetry.

$V = \lambda(\phi + v)^4$ has shift-scale-shift symmetry.

$V = \lambda(\phi_1^2 + \phi_2^2 - v^2)^2$ has $\mathrm{U}(1)$.

Define $\phi_2 = v + \varphi$.

$V = \lambda(\phi_1^2 + \varphi^2 + 2v\varphi)^2$ has shift-$\mathrm{U}(1)$-shift.



LPS method will find symmetry, no matter how broken/hidden it may be.

For example, solve for relationships between c_i in

$$V = c_1 + c_2\phi_1 + c_3\phi_2 + c_4\phi_1^2 + c_5\phi_1\phi_2 + c_6\phi_2^2 + c_7\phi_1^3 + c_8\phi_1^2\phi_2 + c_9\phi_1\phi_2^2 + c_{10}\phi_2^3 + c_{11}\phi_1^4 + c_{12}\phi_1^3\phi_2 + c_{13}\phi_1^2\phi_2^2 + c_{14}\phi_1\phi_2^3 + c_{15}\phi_2^4\,.$$

Spin-1 plus N scalars

$$\mathcal{L} = -\frac{1}{2}\partial^\mu\phi_i\partial_\mu\phi_i - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + J_i A^\mu\partial_\mu\phi_i + K_{ij}A^\mu\phi_i\partial_\mu\phi_j - V(\phi, A^2)$$

General solution for infinitesimals:

$$\begin{aligned}\eta^\mu(x) &= a^\mu + b^\mu{}_\nu x^\nu + c\,x^\mu + 2d_\nu x^\nu x^\mu - d^\mu x^\nu x_\nu \\ \chi_i(x,\phi) &= \alpha_i(x) + \beta_{ij}(x)\phi_j + (2-D)(\tfrac{1}{2}c + d_\nu x^\nu)\phi_i \\ \xi^\mu(x,A) &= \partial^\mu\Lambda(x) + (b^\mu{}_\nu + 2d_\nu x^\mu - 2d^\mu x_\nu)A^\nu + (2-D)(\tfrac{1}{2}c + d_\nu x^\nu)A^\mu\end{aligned}$$

E.g. massive $\mathrm{U}(1)$: when solving rest of determining equations, demand:

- gauge symmetry: $\Lambda(x)$ is arbitrary,
- massive vector: $\frac{\partial V}{\partial A^\mu} = m^2 A_\mu + \ldots$.

$\rightarrow$ derive allowed form of $\mathcal{L}$ and relations between parameters.

1 field: Stückelberg ($J = m$), 2 fields: Higgs.

Automation of LPS method

Large systems lead to an unmanageable set of determining equations.

The LPS method can be cast as a well defined algorithm that completes in finite time, at least up to finding parameter relationships and the rank of the symmetry.

We can construct a computer program which takes in a Lagrangian and returns a list of branches of symmetries and parameter relationships.

1. Compute the determining equations.
   Straightforward.
2. Reduce the determining equations to standard form.
   Algebraically difficult. Includes branching.
3. Compute rank of each branch.
   Simple.

Automation of LPS method

Two (massive) scalars have algebraic determining equation

$$\alpha_1 m_1^2 \phi_1 + \alpha_2 m_2^2 \phi_2 + \beta(m_1^2 - m_2^2)\phi_1\phi_2 = 0\,.$$

Gaussian elimination (with branching) to find null space of

$$\begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_1^2 - m_2^2 \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \beta \end{pmatrix} = 0\,.$$

Differential equations $\rightarrow$ generalised Gaussian elimination.

Define ordering on η^μ and χ_i. Sort terms. Arrange as rows.

Perform "row reduction" to "diagonal" form.

$$c_1(\lambda_i)\,\partial_i f + X_1(f) = 0\,,$$
$$c_2(\lambda_i)\,\partial_{i+j} f + X_2(f) = 0\,.$$

- $c_1(\lambda_i) = 0$: remove $\partial_i f$ term.
- $c_1(\lambda_i) \neq 0$: use $\partial_i f$ to eliminate $\partial_{i+j} f$.

The standard model

Schematic structure of the standard model:

$$\mathcal{L}_{\mathsf{SM}} \sim (\partial\phi)^2 + \phi^2\partial\phi + \phi^2 + \phi^4 + \psi\partial\psi + \phi\psi^2 \, .$$

- $N = 244$ real degrees of freedom (with RH neutrinos and Higgs).
- About 10^7 terms in $\mathcal{L}_{\mathsf{SM}}$.
- Maximum number of determining equations: 2.5×10^6 (but many are duplicated, and many are single term).

Apply the LPS method:

- Find all (continuous) symmetries and *prove* that there are no more.
- Use know values of parameters, and run them.
- Find approximate symmetries.
- Add new degrees of freedom looking for new symmetries (e.g. GUT).
- Given measurements of new particles/interactions, can they form part of a new symmetry?

Handling the size of the standard model

Standard model has many fields, many parameters.

Possible in principle.

In practice the order complexity of the LPS algorithm is too high.

Ways forward:

- One family.
- No colour.
- Semi-numerical approach.

Alternatively, use index notation:

$$\mathcal{L} = S_{ijkl}(\partial_i \phi_j)(\partial_k \phi_l) + T_{ijkl}\phi_i \phi_j (\partial_k \phi_l) + U_{ijk}\psi_i(\partial_j \psi_k) + V(\phi, \psi) \,.$$

Can solve for general symmetries: depends only on *derivative structure*, not on values of S, T, U, or on form of V.

Work in progress . . .

Beyond Lie point symmetries

- Contact symmetries: depend on first derivative of the field.
- Generalised symmetries (Lie-Bäcklund), which allow η^μ and χ_i to depend on arbitrary derivatives of ϕ_i.
- Discrete symmetries (which are not subsets of a continuous group). Hydon, Eur. J. of Appl. Math., 11 (2000) 515. $\rightarrow$ a method to systematically find discrete point symmetries.

Conclusions

Coordinate variation η^μ, field variation χ_i.

Master determining equation:

$$\mathcal{L}\frac{\mathrm{d}\eta^\mu}{\mathrm{d}x^\mu} + \frac{\partial \mathcal{L}}{\partial x^\mu}\eta^\mu + \frac{\partial \mathcal{L}}{\partial \phi_i}\chi_i + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_i)}\left(\frac{\mathrm{d}\chi_i}{\mathrm{d}x^\mu} - \frac{\partial \phi_i}{\partial x^\nu}\frac{\mathrm{d}\eta^\nu}{\mathrm{d}x^\mu}\right) = 0$$

The Lie point symmetry method:

- Counterpart to the Euler-Lagrange equations.
- Finds all possible symmetries.
- Finds all interesting relationships between parameters.
- Works even for spontaneously broken symmetries.
- Can be automated; crucial for large systems.

Future work:

- Find all symmetries of the standard model.
- Allow for discrete symmetries [Hydon (1998)].
- Extend to supersymmetry [Grundland, Hariton, Snobl (2008)].

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Degrees of freedom in the standard model

$N = 244$ real degrees of freedom (with RH neutrinos):

- gauge $=$ 4 real components $\times$ (1 hyp $+$ 3 weak $+$ 8 strong) $=$ 48,
- leptons $=$ 8 real components $\times$ 3 gens $\times$ (ν $+$ e) $=$ 48,
- quarks $=$ 8 real components $\times$ 3 gens $\times$ 3 cols $\times$ (u $+$ d) $=$ 144,
- and Higgs $=$ 2 real components $\times$ weak-doublet $=$ 4.