Mem-brane world model building

Putting the standard model on a domain-wall brane

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THE UNIVERSITY OF MELBOURNE

We are inspired by the Randall-Sundrum warped metric solution.

RS1 is a compact extra dimension: provides a solution to the hierarchy problem – lots of work on this model. Branes are string theory like objects. Warped throats, inflation, dark matter, ...

RS2 is an infinite extra dimension: solves the trapping of low-energy gravity. Not as much interest because it doesn't solve any major problems, just introduces another dimension.

We will pursue RS2 because it seems a natural extension of 3+1 space.

Most work done in collaboration with:
Ray Volkas (Melbourne U) and Rhys Davies (Oxford U).

(Mem-)Brane worlds

Brane worlds

Premise:

- take the standard model and general relativity
- add an *infinite* extra space dimension
- recover the standard model and general relativity at low energies

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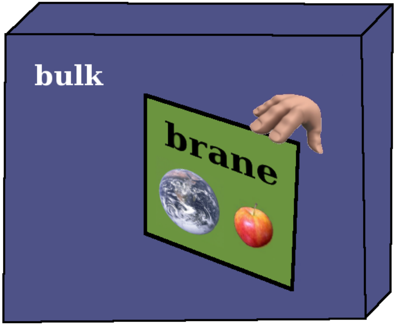

$$\mathcal{S} = \int d^4x \int dy \sqrt{|g|} \left[-M^3 R + \delta(y) \mathcal{L}_{\mathsf{SM}} \right]$$

RS2 model

Need brane and bulk sources:

$$\mathcal{S} = \int d^4x \int dy \left[\sqrt{|g|}(-M^3 R - \Lambda_{\mathsf{bulk}}) + \sqrt{|g^{(4)}|}\delta(y)(\mathcal{L}_{\mathsf{SM}} - \Lambda_{\mathsf{brane}}) \right] \tag{1}$$

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Solve the theory:

- Randall-Sundrum metric ansatz: $ds^2 = e^{-2k|y|} g^{(4)}_{\mu\nu} dx^\mu dx^\nu - dy^2$
- Solve Einstein's equations ($\mathcal{L}_{\text{SM}} = 0$ and $R^{(4)} = 0$):

$$\Lambda_{\text{bulk}} = -12k^2 M^3 \qquad\qquad \Lambda_{\text{brane}} = 12kM^3$$

- Write R in terms of $R^{(4)}$: $R = e^{2k|y|} R^{(4)} - 16k\delta(y) + 20k^2$

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Substitute into (1) and integrate over y:

$$\mathcal{S} = \int d^4x \sqrt{|g^{(4)}|} \left[-\frac{M^3}{k} R^{(4)} + \mathcal{L}_{\mathsf{SM}} \right]$$

A dimensionally reduced theory.

Newton's law

Just need to check Newton's law. Linear tensor fluctuations are:

$$g_{\mu\nu} = e^{-2k|y|}\eta_{\mu\nu} + \sum_n h^{(4)}_{n\ \mu\nu}(x^\mu)\psi_n(y)$$

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Newton's law is modified to:

$$V(r) = -G_N \frac{m_1 m_2}{r}\left(1 + \frac{\epsilon^2}{r^2}\right) \qquad \text{(where } \epsilon = 1/k\text{)}$$

Current experimental bounds are very weak:

$$\epsilon < 12\mu\text{m} \quad \Longrightarrow \quad k > 16 \times 10^{-3}\text{eV}$$

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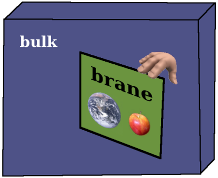
- We have what we wanted: a 5D theory that at low energies looks like our 4D universe.
- But almost no new phenomenology.
- Next step: brane forms naturally.



Thick and smooth RS2

We want to remove the $\delta(y)$ part of the action:

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Geometry, hence gravity, is 5D. So why not try to make all fields 5D?

- First we show how to make a dynamical brane.
- Then we show how to trap scalars, fermions and gauge fields to the brane.
- Finally we present a 4+1-d $SU(5)$ based extension to the standard model.

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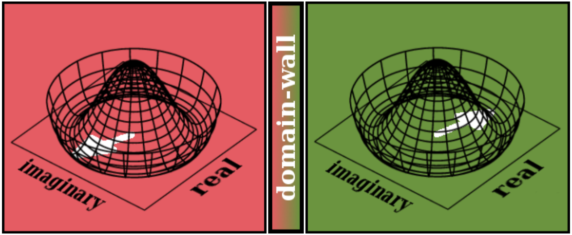
Turn off warped gravity for now – just think about trapping 5D fields.

A domain-wall as a brane

Idea: imagine the Higgs VEV had one value here, another there.

The interface is a *domain-wall*.

$$V = \lambda(\phi^*\phi - v^2)^2$$



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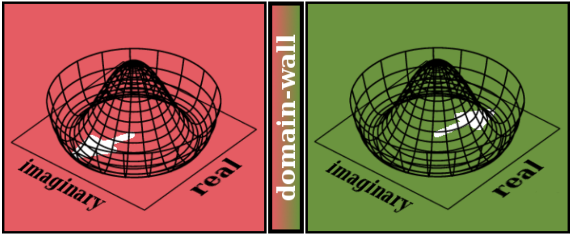
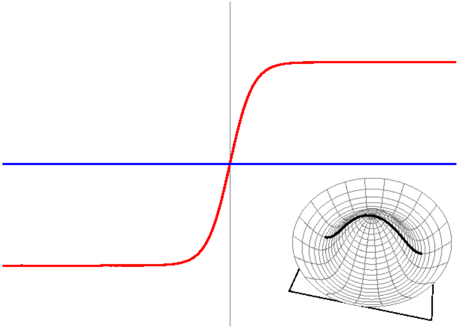
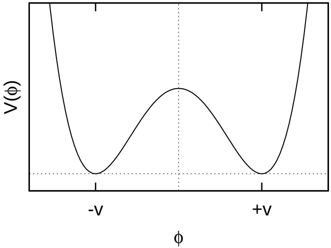




This is unstable – vacua can be *continuously* deformed to each other.

(key: real,imaginary)

Disconnected degenerate vacua



We need a potential with *disconnected* and *degenerate* vacua:

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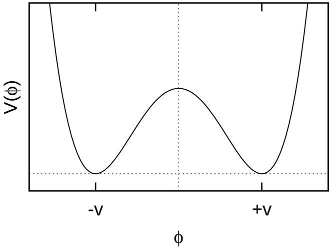
with ϕ now a scalar.

Disconnected degenerate vacua



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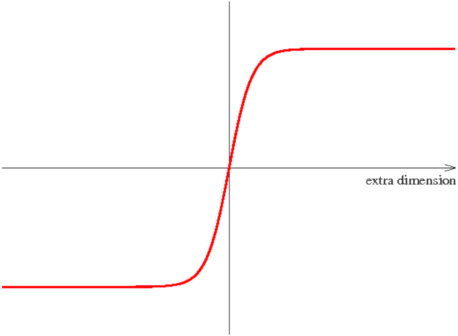
Lagrangian for $\phi(x^\mu, y)$:

$$\mathcal{L} = \frac{1}{2}\partial_M \phi \, \partial^M \phi - V(\phi)$$

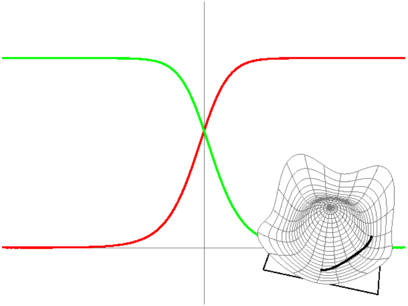
A solution is the *kink*:

$$\phi(y) = v \tanh(\sqrt{2\lambda} v y)$$

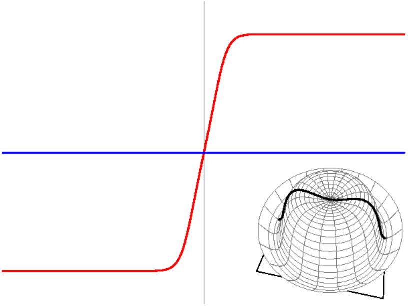
It is stable!



These examples use two scalar fields to form the wall.



$$V = \lambda_1(\phi_1^2 + \phi_2^2 - v^2)^2 + \lambda_2\phi_1^2\phi_2^2$$



$$V = \lambda(\phi_1^2+\phi_2^2-v_1^2)^2(\phi_1^2+\phi_2^2+v_2^2)$$

More complicated domain-walls

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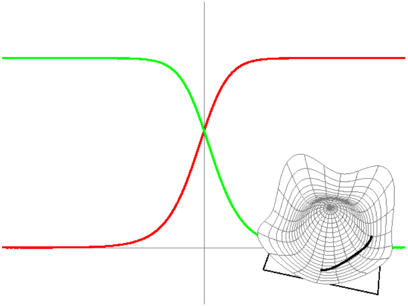

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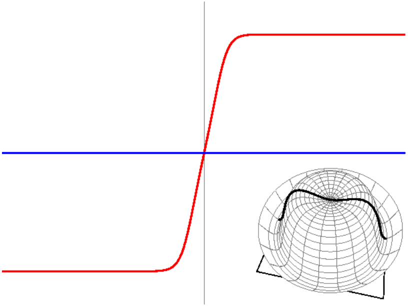

$V = \lambda(\phi_1^2+\phi_2^2-v_1^2)^2(\phi_1^2+\phi_2^2+v_2^2)$

In a realistic model, symmetries dictate V.

To determine stability, expand ϕ in normal modes about the background: $\phi(x,t) = \phi_{\mathsf{bg}}(x) + \sum_n \xi_n(x)e^{i\omega_n t}$. Make sure $\omega_n^2 \geq 0$

Trapping matter fields

Trapping scalar fields

Aim: to trap a 5D scalar field $\Xi(x^\mu, y)$ to the brane.

A simple quartic coupling works:

$$\mathcal{S} = \int d^4x \int dy \left[\frac{1}{2}\partial^M \phi \, \partial_M \phi - V(\phi) + \frac{1}{2}\partial^M \Xi \, \partial_M \Xi - W(\Xi) - g\phi^2 \Xi^2 \right]$$

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Expand Ξ in extra dimensional (Kaluza-Klein) modes:

$$\Xi(x^\mu, y) = \sum_n \xi_n(x^\mu) k_n(y)$$

ξ_n are the 4D fields, k_n their extra-dimensional profile. The profiles satisfy a Schrödinger equation:

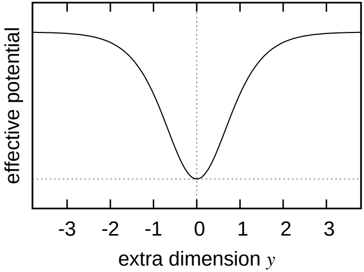
$$\left(-\frac{d^2}{dy^2} + 2g\phi^2_{\mathsf{bg}}\right) k_n(y) = E_n^2 k_n(y)$$

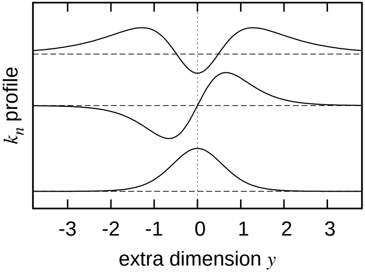
The energy eigenvalues E_n are related to the mass of the 4D field ξ_n.

Trapping via a potential well

The effective potential acts like a well.

$$\left(-\frac{d^2}{dy^2} + 2g\phi_{\text{bg}}^2\right) k_n = E_n^2 k_n$$





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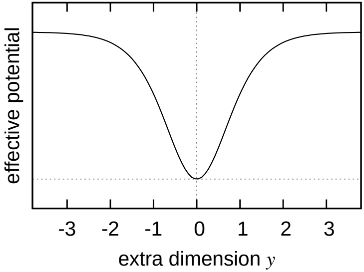

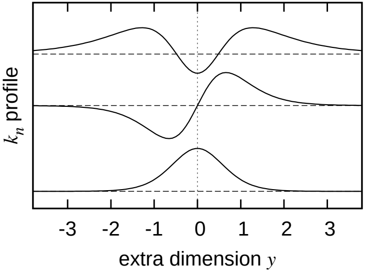

To get 4D theory, substitute mode expansion into action and integrate y:

$$\mathcal{S} = \int d^4x \left[\sum_n \left(\frac{1}{2} \partial^\mu \xi_n \partial_\mu \xi_n - m_n^2 \xi_n^2 \right) + \text{(higher order terms)} \right]$$

- Orthonormal basis $k_n \implies$ diagonal kinetic and mass terms.
- m_n can be tuned.

Trapping fermions

We can trap a fermion $\Psi(x^\mu, y)$ to the brane with a Yukawa coupling:

$$\mathcal{S} = \int d^4x \int dy \left[\frac{1}{2}\partial^M\phi\, \partial_M\phi - V(\phi) + \overline{\Psi} i\Gamma^M \partial_M \Psi - h\phi\overline{\Psi}\Psi \right]$$

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Decompose into left- and right-chiral fields and Kaluza-Klein modes:

$$\Psi(x^\mu, y) = \sum_n \left[\psi_{Ln}(x^\mu) f_{Ln}(y) + \psi_{Rn}(x^\mu) f_{Rn}(y) \right]$$

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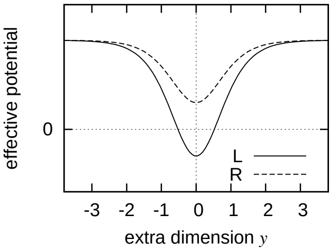
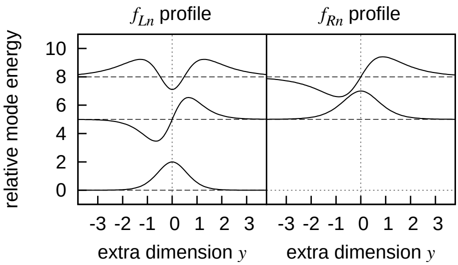
Schrödinger equation (mode index n suppressed):

$$\left(-\frac{d^2}{dy^2} + (h^2\phi_{\text{bg}}^2 \mp h\phi'_{\text{bg}})\right) f_{L,R}(y) = m^2 f_{L,R}(y)$$





Gravity and matter fields

Brane (domain-wall/kink), trapped scalar and fermion. Plus gravity:

$$\mathcal{S} = \int d^4x \int dy \sqrt{|g|} \Big[-M^3 R - \Lambda_{\mathsf{bulk}} + \frac{1}{2}\partial^M \phi \, \partial_M \phi - V(\phi) + \frac{1}{2}\partial^M \Xi \, \partial_M \Xi - W(\Xi) - g\phi^2 \Xi^2 + \overline{\Psi} i \Gamma^M \partial_M \Psi - h\phi \overline{\Psi}\Psi \Big]$$

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Dimensionally reduce by integrating over y:

$$\mathcal{S} = \int d^4x \sqrt{|g^{(4)}|} \Big[-M_{\text{4D}}^2 R^{(4)} + \text{(brane dynamics)} + \frac{1}{2}\partial^\mu \xi_n \partial_\mu \xi_n - m_n^2 \xi_n^2 - \tau_{mnop}\xi_m\xi_n\xi_o\xi_p - \text{(brane interactions)} + \overline{\psi}_{L0} i\gamma^\mu \partial_\mu \psi_{L0} + \overline{\psi}_n (i\gamma^\mu\partial_\mu - \mu_n)\psi_n - \text{(brane interactions)} \Big]$$

4D parameters (M_{4D}, m_n, τ_{mnop}, μ_n, brane dynamics) determined by eigenvalue spectra and overlap integrals.

Warped matter

Warped metric $ds^2 = e^{-2\sigma(y)}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2$ modifies profile equation:

$$\left(-\frac{d^2}{dy^2} + 5\sigma'\frac{d}{dy} + 2\sigma'' - 6\sigma'^2 + U(y)\right) f_{Ln}(y) = m_n^2 e^{2\sigma} f_{Ln}(y)$$

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Conformal coordinates $ds^2 = e^{-2\sigma(y(z))}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2)$.
Rescale $f_{Ln}(y) = e^{2\sigma}\tilde{f}_{Ln}(z)$:

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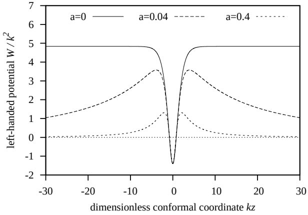
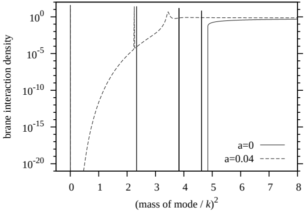
Matter trapping potentials are warped down.

- Finite bound state lifetimes.
- Resonances.
- Tiny probability of interaction with continuum.





($a \sim 1/M^3 \sim$ 5D Newton's constant)

Trapping gauge fields

Confining gauge fields

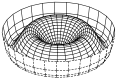
Need to trap gauge fields or e.g. Coulomb potential would be $V_{\text{Coulomb}} \sim 1/r^2$.

Not as simple as a Kaluza-Klein mode expansion:

- Photon and gluons *must* remain massless.
- Need to preserve gauge universality at 3+1-d level.

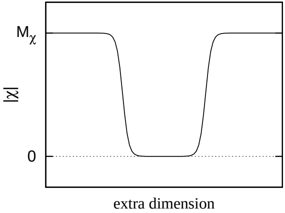
We use the Dvali-Shifman mechanism, following an argument due to arXiv:0710.5051 (Dvali et al).

Abelian Higgs model



$U(1)$ gauge theory, charged Higgs χ:

$$\mathcal{S} = \int d^4x \int dy \left[\frac{-1}{4g^2} F^{MN} F_{MN} + \frac{1}{2} (D^M \chi)^\dagger D_M \chi - (|\chi|^2 - M_\chi^2)^2 \frac{|\chi|^2}{M_\chi^2} \right]$$



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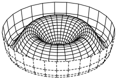

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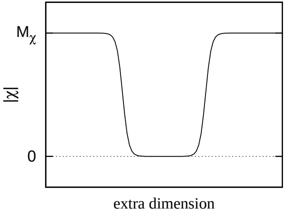
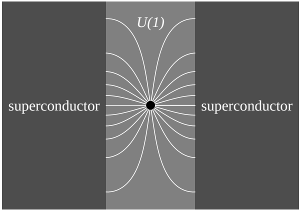




In the bulk:

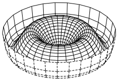
- $U(1)$ is broken, massive photon $\sim M_\chi$.
- Higgs vacuum is a superconductor.
- Electric charges are screened.

On the brane:

- $U(1)$ is restored, massless photon.
- Electric field ends on Higgs vacuum.

Charge screening leaks onto the brane!

Using a dual superconductor



$SU(2)$ gauge theory, adjoint Higgs χ^a $(a = 1, 2, 3)$:

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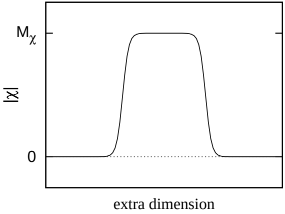

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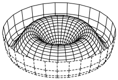

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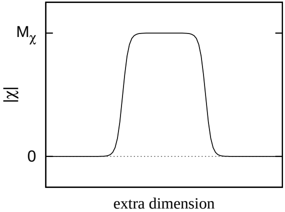

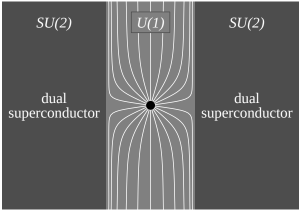

In the bulk:

- $SU(2)$ is restored, in confining regime.
- Large mass gap $\sim M_\chi$ to colourless state.
- QCD-like vacuum is dual superconductor.

On the brane:

- $SU(2)$ broken to $U(1)$, massless photon.
- Electric field repelled from dual superconductor.

For distances much larger than brane width, electric potential $\sim 1/r$.

Dvali-Shifman model

Stabilise the domain-wall with an extra uncharged scalar field η:

$$\mathcal{S} = \int d^4x \int dy \Big[\frac{-1}{4g^2} G^{aMN} G^a_{MN} + \frac{1}{2}\partial^M \eta \partial_M \eta + \frac{1}{2}(D^M \chi^a)^\dagger D_M \chi^a$$
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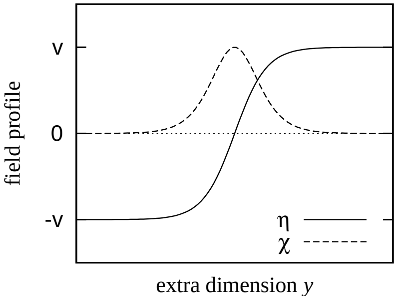
- η has a kink profile.
- If $\kappa^2 - v^2 < 0$, χ becomes tachyonic near domain-wall (where $\eta \sim 0$).
- True vacuum has $\chi \neq 0$ near domain-wall.
- χ breaks symmetry near wall and confines gauge fields.



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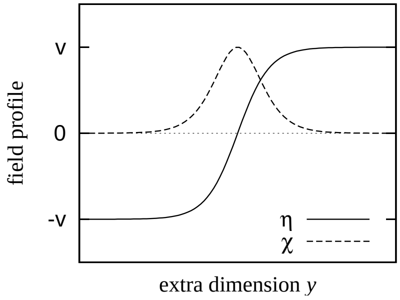

Can add gravity: self consistently solve σ (warped metric profile), η, χ.

The Dvali-Shifman mechanism:

- Works with any non-Abelian $SU(N)$ theory.
- Assumes the $SU(N)$ theory is confining (not proven for 5D).
- Has gauge universality:
  - Charges in the bulk are connected to the brane by a flux tube.
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Obvious choice for $SU(N)$ group is $SU(5)$.

Aside: The standard model and $SU(5)$

Quantum numbers of the standard model

Representations under $SU(3) \times SU(2)_L \times U(1)_Y$:

$$q_L \sim (\mathbf{3}, \mathbf{2})_{1/3} \qquad u_R \sim (\mathbf{3}, \mathbf{1})_{4/3} \qquad d_R \sim (\mathbf{3}, \mathbf{1})_{-2/3}$$

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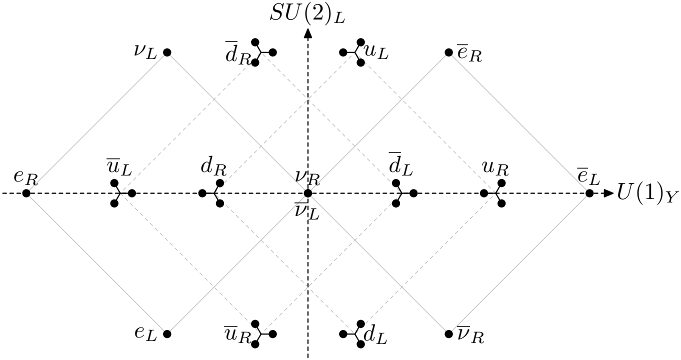
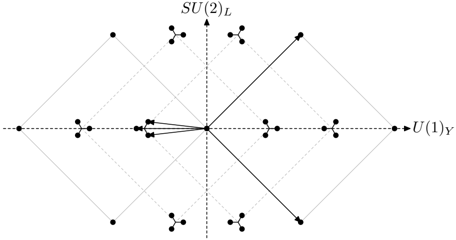
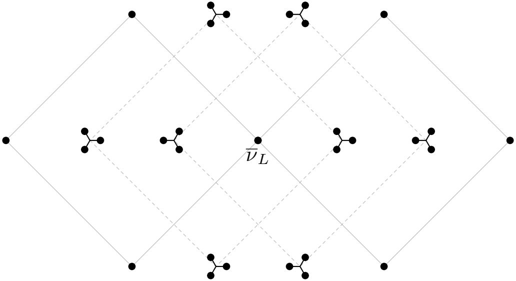
A 4D combined weight diagram:



Spanning the diagram

With the right basis vectors, we can span this space ($\mathbb{Z}_2$ coefficients).





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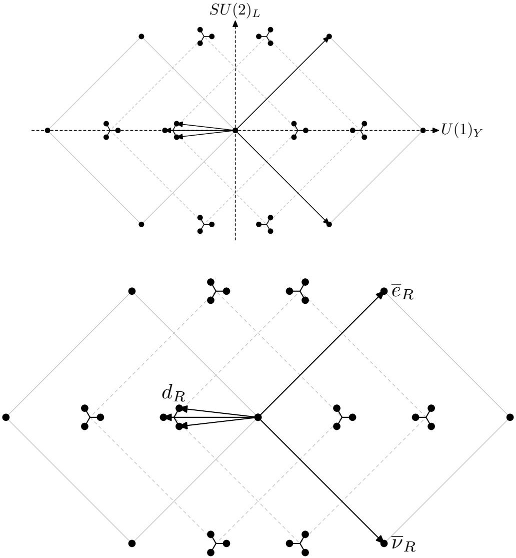

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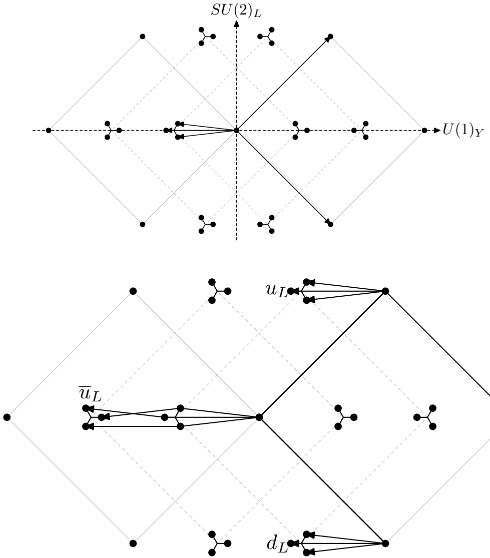

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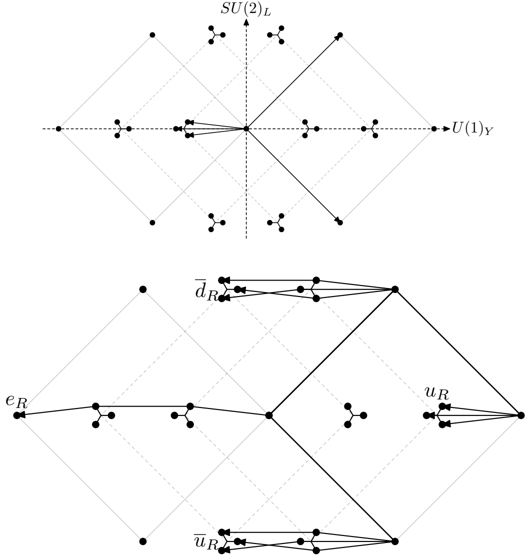

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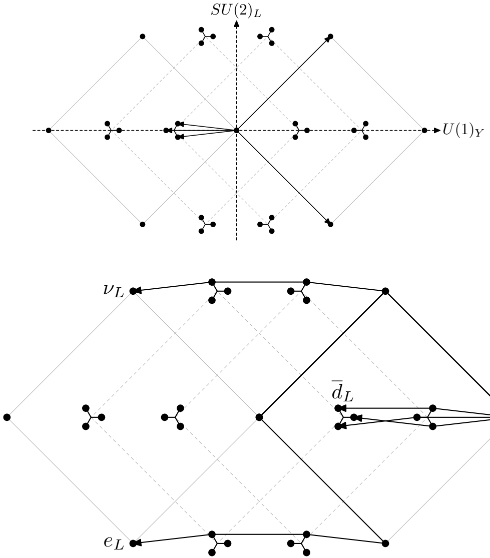

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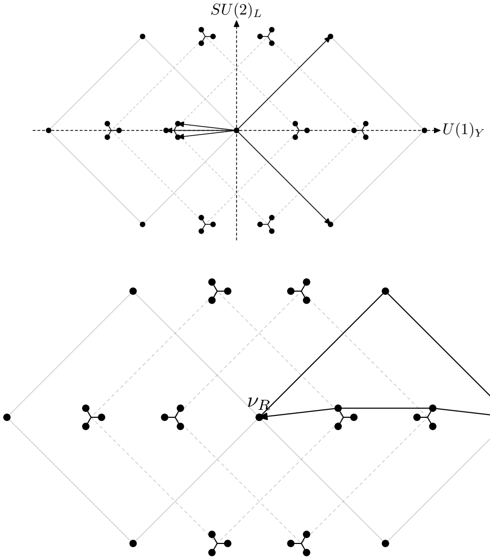
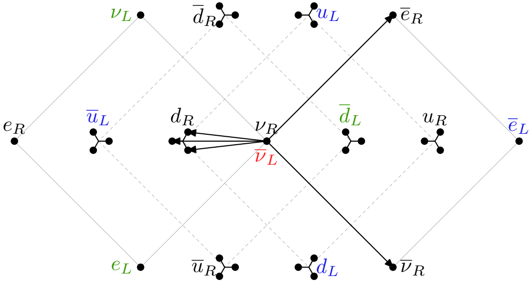


Multiplets of $SU(5)$

Take the basis vectors as the $\mathbf{5}$ of $SU(5)$:



Anti-symmetric products of the $\mathbf{5}$ give:

		$\mathbf{1}$	$\bar{\nu}_L$
		$\mathbf{5}^*$	$\bar{d}_L^{r,w,b}$ ν_L e_L
$(\mathbf{5}\times\mathbf{5})_A$	$=$	$\mathbf{10}$	$\bar{u}_L^{r,w,b}$ $u_L^{r,w,b}$ $d_L^{r,w,b}$ $\bar{e}_L$

Putting it all together

The $SU(5)$ model

Want the standard model on the brane: $SU(3) \times SU(2)_L \times U(1)_Y$.

Dvali-Shifman needs a *larger* gauge group in the bulk:

$SU(5)$ is a perfect fit!

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Higgs doublet goes in a $\mathbf{5}^*$.

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Summary:

- $4+1$-dimensional theory – all spatial dimensions the same.
- $SU(5)$ local gauge symmetry, $\mathbb{Z}_2$ discrete symmetry.
- Field content:
  - gauge fields: $G_{MN} \sim \mathbf{24}$.
  - scalars: $\eta \sim \mathbf{1}$, $\chi \sim \mathbf{24}$, $\Phi \sim \mathbf{5}^*$.
  - fermions: $\Psi_5 \sim \mathbf{5}^*$, $\Psi_{10} \sim \mathbf{10}$.
- The standard model emerges as a low energy approximation.

Ignore gravity for now.

The theory is described by:

$$\begin{aligned}\mathcal{S} = \int d^4x \int dy \Big[& \frac{-1}{4g^2} G^{aMN} G^a_{MN} + \frac{1}{2}\partial^M \eta \partial_M \eta + \mathrm{Tr}\left((D^M \chi)^\dagger (D_M \chi) \right) \\ & + (D^M \Phi)^\dagger (D_M \Phi) + \overline{\Psi}_5 i \Gamma^M D_M \Psi_5 + \overline{\Psi}_{10} i \Gamma^M D_M \Psi_{10} \\ & - h_{5\eta} \overline{\Psi}_5 \Psi_5 \eta - h_{5\chi} \overline{\Psi}_5 \chi^T \Psi_5 \\ & \qquad - h_{10\eta} \mathrm{Tr}(\overline{\Psi}_{10} \Psi_{10}) \eta + 2 h_{10\chi} \mathrm{Tr}(\overline{\Psi}_{10} \chi \Psi_{10}) \\ & - h_- \overline{(\Psi_5)^c} \Psi_{10} \Phi - h_+ (\epsilon \overline{(\Psi_{10})^c} \Psi_{10} \Phi^*) + \text{h.c.} \\ & - (c\eta^2 - \mu_\chi^2) \mathrm{Tr}(\chi^2) - d\eta \, \mathrm{Tr}(\chi^3) \\ & \qquad - \lambda_1 \left[\mathrm{Tr}(\chi^2) \right]^2 - \lambda_2 \mathrm{Tr}(\chi^4) - l(\eta^2 - v^2)^2 \\ & - \mu_\Phi^2 \Phi^\dagger \Phi - \lambda_3 (\Phi^\dagger \Phi)^2 - \lambda_4 \Phi^\dagger \Phi \eta^2 \\ & \qquad - 2\lambda_5 \Phi^\dagger \Phi \, \mathrm{Tr}(\chi^2) - \lambda_6 \Phi^\dagger (\chi^T)^2 \Phi - \lambda_7 \Phi^\dagger \chi^T \Phi \eta \Big]\end{aligned}$$

with kinetic, brane trapping, mass and Dvali-Shifman terms.

Split fermions

Let Ψ_{nY} be the components of Ψ_5 and Ψ_{10} ($n = 5, 10$, Y = hypercharge of component), e.g. $\Psi_5 \supset \Psi_{5,-1} = l_L$. Dirac equation:

$$\left[i\Gamma^M \partial_M - h_{n\eta}\eta(y) - \sqrt{\frac{3}{5}}\frac{Y}{2}\, h_{n\chi}\chi_1(y) \right] \Psi_{nY}(x^\mu, y) = 0$$

Each Ψ_{nY} is a non-chiral 5D field: need to extract the confined left-chiral zero-mode (recall the mode expansion and Schrödinger equation approach):

$$\Psi_{nY}(x^\mu, y) = \psi_{nY,L}(x^\mu) f_{nY}(y) + \text{massive modes}$$

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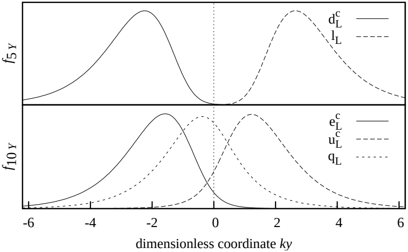
The effective Schrödinger potential depends on Y.

Thus each component $\psi_{nY,L}$ has a different profile f_{nY}.



Split Higgs

Φ contains the Higgs doublet Φ_w and a coloured triplet Φ_c. Mode expand $\Phi_{w,c}(x^\mu, y) = \phi_{w,c}(x^\mu) p_{w,c}(y)$. Schrödinger equation for $p_{w,c}$ is:

$$\left(-\frac{d^2}{dy^2} + \frac{3Y^2}{20}\lambda_6\chi_1^2 + \sqrt{\frac{3}{5}}\frac{Y}{2}\lambda_7\eta\chi_1 + \dots\right) p_{w,c}(y) = m_{w,c}^2 p_{w,c}(y)$$

Critical that ground states have:

- $m_w^2 < 0$ to break electroweak symmetry.
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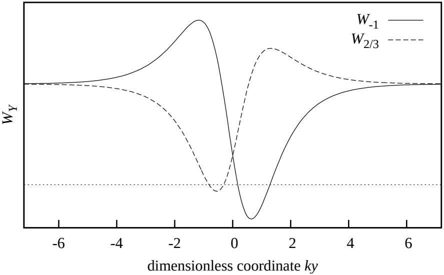
Large enough parameter space to allow this.



Features

Standard model parameters are computed from overlap integrals.

With one generation of fermions, parameters are easy to fit.

The model overcomes the major $SU(5)$ obstacles:

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- Coloured Higgs induced proton decay is suppressed.
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Adding gravity:

- Solve for warped metric, kink and Dvali-Shifman background.
- Continuum fermion and scalar modes are highly suppressed on the brane.
- Main features remain.

Future work and extensions

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- Understand confinement of $SU(N)$ in 5D.
- Three families with full parameter fitting.
- Neutrino masses and mixings.
- Brane cosmology.

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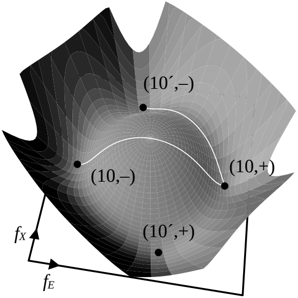
One promising extension is to the E_6 group:

- $E_6 \to SO(10)$ in the bulk.
- $SO(10) \to SU(5)$ on the brane due to clash-of-symmetries and Dvali-Shifman.
- Can eliminate kink scalar field η.
- Can unify Ψ_5 and Ψ_{10}.
- Large reduction of free parameters.



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- A. Davidson, D.P. George, A. Kobakhidze, R.R. Volkas and K.C. Wali
  $SU(5)$ *grand unification on a domain-wall brane from an* E_6*-invariant action*
  arXiv:0710.3432