Fake supergravity and domain-wall models with a soft wall

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Nikhef Theory Center Meeting — 18th June 2010

Based on arXiv:1006.2827, with Mert Aybat

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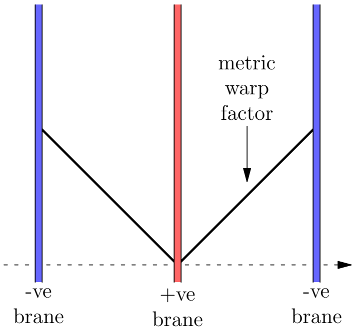
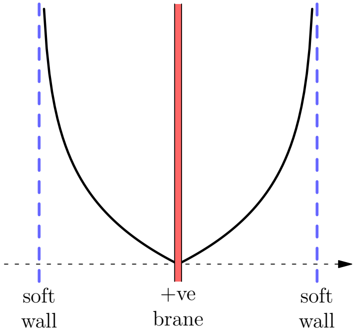
Compact extra dimensions

Physics beyond the standard model: *compact* extra dimensions.





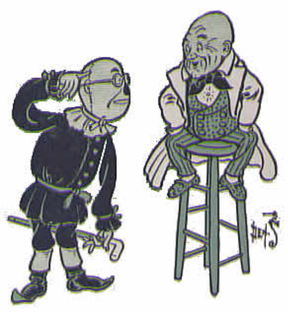
Randall-Sundrum hard wall → RS soft-wall models.

Original soft-wall motivation: AdS/QCD and linear Regge trajectories.

Karch, Katz, Son & Stephanov, PRD74, 015005 (2006)

Now exist early BSM models.

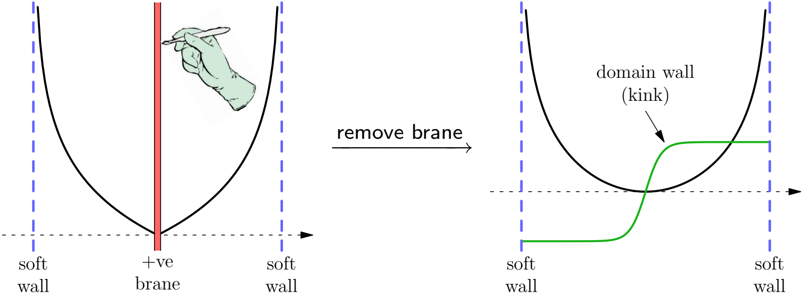
Batell & Gherghetta, PRD78, 026002 (2008), Falkowski & Perez-Victoria, JHEP 12, 107 (2008),
Batell, Gherghetta & Sword, PRD78, 116011 (2008), Cabrer, von Gersdorff & Quiros, arXiv:0907.5361,
Aybat & Santiago, PRD80, 035005 (2009).



"Can't you give me brains?" asked the Scarecrow.

"You don't need them!" replied the Wizard.

Our aim is to replace the brane with a domain wall.
$\longrightarrow$ *Must ensure stability.*



General set-up

General framework: GR with N scalar fields:

$$\mathcal{S} = \int d^4x\, dy \left[\sqrt{-g}\left(M^3 R + \mathcal{L}_{\text{matter}}\right) - \sqrt{-g_4}\lambda\right] .$$

where

$$\mathcal{L}_{\text{matter}} = -\frac{1}{2}\sum_i g^{MN}\partial_M\Phi_i\partial_N\Phi_i - V(\{\Phi_i\}) ,$$

$$\lambda = \lambda(\{\Phi_i\}) = \sum_\alpha \lambda_\alpha(\{\Phi_i\})\delta(y - y_\alpha) .$$

The governing equations of this theory are:

- Einstein's equations: $G_{MN} = \frac{1}{2M^3}T_{MN}$.
- Euler-Lagrange: $\partial_M\left(\sqrt{-g}g^{MN}\partial_N\Phi_i\right) = \sqrt{-g}V_i + \sqrt{-g_4}\lambda_i$.

Very generic, many applications.

Background configuration

Background configuration is a *static* solution to the equations of motion.

Ansatz is:

$$ds^2 = e^{-2\sigma(y)}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2\,, \qquad \Phi_i(x^\mu, y) = \phi_i(y)\,.$$

Equations are:

$$3\sigma'' = \frac{1}{2M^3}\left(\sum_i \phi_i'^2 + \lambda(\{\phi_j\})\right)\,,$$

$$12\sigma'^2 - 3\sigma'' = -\frac{1}{2M^3}\left[2V\left(\{\phi_j\}\right) + \lambda\left(\{\phi_j\}\right)\right]\,,$$

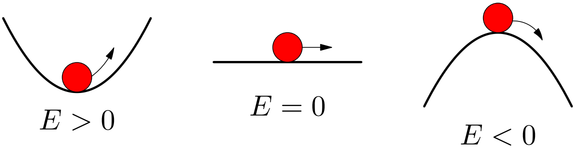
$$\phi_i'' - 4\sigma'\phi_i' - V_i(\{\phi_j\}) - \lambda_i(\{\phi_j\}) = 0\,.$$

(where $\sigma' = d\sigma/dy$, $V_i = \partial V/\partial\Phi_i$ etc.)

$\{V$, λ, integration constants$\}$ define a configuration.
Is it stable in the space of configurations?

Perturbative stability

Perturb the background configuration → eigenvalue problem.



Spin-0 and spin-2 perturbations described by:

$$ds^2 = e^{-2\sigma}\left[(1 - 2F(x^\mu, y)\eta_{\mu\nu} + h_{\mu\nu}(x^\mu, y)\right] dx^\mu dx^\nu + \left[1 + 4F(x^\mu, y)\right] dy^2,$$
$$\Phi_i(x^\mu, y) = \phi_i(y) + \varphi_i(x^\mu, y)\ .$$

(Axial gauge $h_{\mu 5} = 0$, transverse traceless $\partial^\mu h_{\mu\nu} = \eta^{\mu\nu} h_{\mu\nu} = 0$.)

Degrees of freedom:

- Spin-2: $h_{\mu\nu}$ decouples from F and φ_i. Is non-tachyonic. Has a zero mode (4D graviton). Known RS2 result.
- Spin-0: Non-trivial. Physical modes are mixtures of F and φ_i.

Spin-0 perturbations

For spin-0, the equations to solve are ($\Box = \partial^\mu \partial_\mu$):

$$6M^3(F' - 2\sigma' F) = \phi_i' \varphi_i \,,$$
$$6M^3(-e^{2\sigma}\Box F - 2\sigma' F' + F'') = 2\phi_i' {\varphi_i}' + 2\, \lambda|_{\mathsf{bg}}\, F + \lambda_i|_{\mathsf{bg}}\, \varphi_i \,,$$
$$e^{2\sigma}\Box\varphi_i + {\varphi_i}'' - 4\sigma' {\varphi_i}' - 6\phi_i' F' - \, (4V_i + 2\lambda_i)|_{\mathsf{bg}}\, F - \, (V_{ij} + \lambda_{ij})|_{\mathsf{bg}}\, \varphi_j = 0.$$

We can do it! Go to conformal coordinates ($dy = e^{-\sigma} dz$), rescale fields:

$$F(y) = \tfrac{1}{\sqrt{12}} e^{3\sigma/2} \chi(z(y)) \,, \qquad \varphi_i(y) = M^{3/2} e^{3\sigma/2} \psi_i(z(y)) \,.$$

Massage into a familiar form:

$$-\chi'' + (\mathcal{V}_{00} + \mathcal{B}_{00})\chi + (\mathcal{V}_{0i} + \mathcal{B}_{0i})\psi_i = \Box\chi \,,$$
$$-{\psi_i}'' + (\mathcal{V}_{0i} + \mathcal{B}_{0i})\chi + (\mathcal{V}_{ij} + \mathcal{B}_{ij})\psi_j = \Box\psi_i \,.$$

where $\mathcal{B}_{mn}$ contain brane-only terms, and

$$\mathcal{V}_{00} = \tfrac{9}{4}\sigma'^2 + \tfrac{5}{2}\sigma'' \,, \qquad \mathcal{V}_{0i} = \tfrac{2}{\sqrt{3M^3}} \phi_i'' \,,$$
$$\mathcal{V}_{ij} = \left(\tfrac{9}{4}\sigma'^2 - \tfrac{3}{2}\sigma''\right) \delta_{ij} + \tfrac{1}{M^3} \phi_i' \phi_j' + e^{-2\sigma}\, V_{ij}|_{\mathsf{bg}} \,,$$

Fake supergravity

We now specialise to potentials $V(\{\Phi_i\})$ generated using the fake supergravity approach (no branes):

$$V(\{\Phi_i\}) = \sum_i \frac{1}{2} \left[W_i(\{\Phi_i\})\right]^2 - \frac{1}{3M^3} \left[W(\{\Phi_i\})\right]^2 \ .$$

DeWolfe, Freedman, Gubser & Karch, PRD62, 046008 (2000)
Freedman, Nunez, Schnabl & Skenderis, PRD69, 104027 (2004)

For any W, solutions to Einstein's and Euler-Lagrange are:

$$\sigma' = \tfrac{1}{6M^3} W(\{\phi_i\}) \ ,$$
$$\phi_i' = W_i(\{\phi_i\}) \ .$$

This is a set of *first order* equations:

- W encodes for V and half of the integration constants.
- Can take $\sigma(y_0) = 0$ without loss of generality.
- Have N integration constants left: set of values $\{\phi_i(y_0)\}$.

$\{W, \phi_i(y_0)\}$ uniquely define a configuration. Is it stable?

Perturbative stability in fake SUGRA approach

Recall, for general V perturbations obey

$$-\begin{pmatrix}\chi \\ \psi_i\end{pmatrix}'' + \begin{pmatrix}\mathcal{V}_{00} & \mathcal{V}_{0j} \\ \mathcal{V}_{0i} & \mathcal{V}_{ij}\end{pmatrix}\begin{pmatrix}\chi \\ \psi_j\end{pmatrix} = \Box\begin{pmatrix}\chi \\ \psi_i\end{pmatrix} .$$

Using fake SUGRA, we find that $\mathcal{V} = \mathcal{S}^2 + \mathcal{S}'$, where

$$\mathcal{S} = e^{-\sigma}\left.\begin{pmatrix}\frac{1}{12M^3}W & \frac{1}{\sqrt{3M^3}}W_j \\ \frac{1}{\sqrt{3M^3}}W_i & -\frac{1}{4M^3}\delta_{ij}W + W_{ij}\end{pmatrix}\right|_{\text{bg}} .$$

Write $\Psi = (\chi, \psi_i)^T$. Perturbations obey

$$(\partial_z + \mathcal{S})(-\partial_z + \mathcal{S})\Psi = \Box\Psi .$$

Fourier transform on x^μ, multiply by $\Psi^\dagger$ on left and integrate:

$$\int |(-\partial_z + \mathcal{S})\Psi|^2\, dz + (\text{boundary terms}) = E\int |\Psi|^2 dz .$$

Boundary terms vanish for warped metric. **We find $E \geq 0$.**

Mini summary

Our framework contains GR, N scalars and branes.

Fake SUGRA:

- A restriction on the scalar potential V.
- Gives first order equations for background.
- Without branes, there are no tachyonic perturbations, $E \geq 0$.

What about zero modes, $E = 0$?

They are important: zero modes can correspond to changes in the size of the compact extra dimension.

To stabilise a model, we must ensure there are no zero modes.

$N = 1$ systems

With $N = 1$ scalar, can use constraint Einstein equation to eliminate ψ_1 in terms of χ. Then redefine $\chi = \mathcal{S}_{01} g$.

Schrödinger-like equation for g is factorisable:

$$(\partial_z - \mathcal{S}_{11})(-\partial_z - \mathcal{S}_{11})g + \mathcal{S}_{01}^2 g = Eg \, .$$

(Recall: $\mathcal{S}_{01} = \frac{1}{\sqrt{3M^3}} W_1$, $\mathcal{S}_{11} = -\frac{1}{4M^3} W + W_{11}$.)

Multiply by g and integrate:

$$\int |(-\partial_z - \mathcal{S}_{11})g|^2 \, dz + \int |\mathcal{S}_{01} g|^2 + (\text{boundary terms}) = E \int |g|^2 dz \, .$$

For warped metric, boundary terms vanish. Then for $E = 0$ require:

- $(-\partial_z - \mathcal{S}_{11})g = 0$.
- $\mathcal{S}_{01} g = 0$.

Systems with $N = 1$ do not have a zero mode.

$N = 2$ zero-mode theorem

For $N = 2$ there may or may not be a zero mode.

Theorem: *For a system of definite parity with N scalar fields that couple to gravity, the number of independent normalisable zero modes with $E = 0$ is at most equal to the number of fields whose background solutions are even.*

Proof: If a zero mode exists, adding it to the background takes you to another background, generated using the same superpotential but with different integration constants. (Recall: $\sigma' = \frac{1}{6M^3}W$, $\phi_i' = W_i$.)

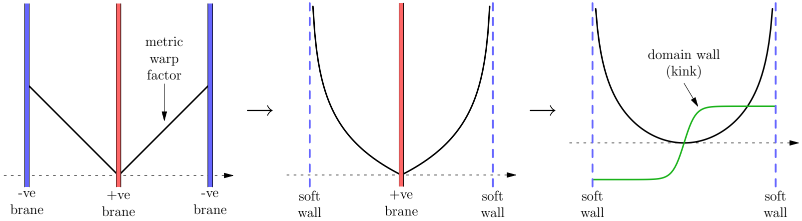
Zero modes $\leftrightarrow$ available integration constants.

No integration constants $\implies$ no zero modes.

Look at some examples.

Domain-wall models with a soft wall

Branes, soft walls and domain walls.



Our domain-wall model is specified by

$$\mathcal{S} = \int d^4x\, dy \sqrt{-g} \left[M^3 R - \sum_{i=1,2} \tfrac{1}{2} g^{MN} \partial_M \Phi_i \partial_N \Phi_i - \sum_{i=1,2} \tfrac{1}{2} W_i^2 + \tfrac{1}{3M^3} W^2 \right].$$

- Φ_1, the dilaton: diverges at finite y to create a soft wall.
- Φ_2, the domain wall: has a kink profile to provide energy density at the origin.
- σ, the warp factor: diverges at finite y.

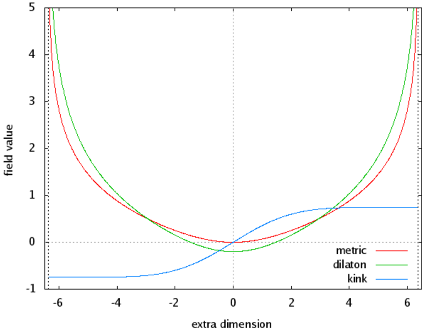
Example $N = 2$ even system — unstable

Φ_1 is even, Φ_2 is odd, σ is even.

Superpotential: $W(\Phi_1, \Phi_2) = e^{\nu\Phi_1}\left(a\Phi_2 - b\Phi_2^3\right)$

Φ_1 is even:

- choice of integration constant
- corresponding zero mode
- size of extra dimension is *not* fixed



$(\nu = 1.4,\ a = 0.5,\ b = 0.3)$

For this case, we have found the explicit solution for the zero mode.

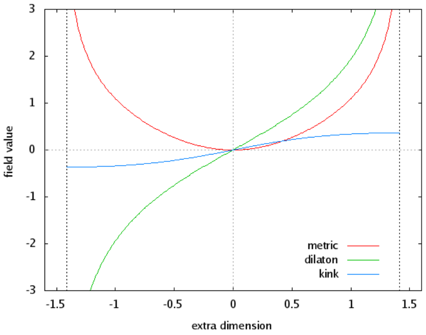
Example $N = 2$ odd system — stable

Φ_1 is odd, Φ_2 is odd, σ is even.

Superpotential: $W(\Phi_1, \Phi_2) = \alpha \sinh(\nu \Phi_1) + \left(a\Phi_2 - b\Phi_2^3\right)$

Φ_1 and Φ_2 are odd:

- no integration constants to choose
- background solution is unique
- no zero modes
- size of extra dimension fixed by parameters in W

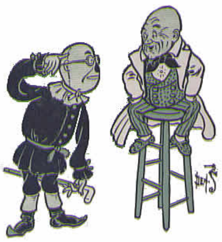


$(\alpha = 1,\ \nu = 1.4,\ a = 0.5,\ b = 0.3)$

An example of a domain-wall model with a soft wall, that stabilises the size of a compact extra dimension.

Summary and future work

Branes: not necessary!



We can have a stable, compact extra dimension:

- Soft-wall at edge of space; replaces negative brane.
- Domain-wall at origin; replaces positive brane.
- Additional scalar (dilaton) cuts off space.

Lesson: using fake SUGRA, need definite parity and all scalars must be odd to eliminate zero modes.

Technical questions:

- No fake SUGRA: can we have even fields?
- Odd fields, but not definite parity: are there zero modes?
- Any applicability to AdS/QCD correspondence?

Most interesting questions:

- Can we solve the hierarchy problem?
- Can we build a realistic standard model?